

WIND TURBINE POWER, ENERGY, AND TORQUE

Who has gathered the wind in his fists? Proverbs 30:4

Gathering or harvesting the wind has been of concern to man for a long time. As mentioned earlier, wind turbines have been used for several centuries and literally millions of units have been put into service. For the most part, these machines performed their intended purpose well, and in many cases were still being used with minimum maintenance after half a century of service. Operational machines four centuries old are not unheard of, pointing out the fact that planned obsolescence does not have to be a part of engineering work. A major reason for their success was the lack of competition, of course. There was a choice of using the wind to perform some task or doing it by hand, and doing it by hand is last choice for most people.

Today, wind turbines have to compete with many other energy sources. It is therefore important that they be cost effective. They need to meet any load requirements and produce energy at a minimum cost per dollar of investment. Performance characteristics such as power output versus wind speed or versus rotor angular velocity must be optimized in order to compete with other energy sources. Yearly energy production and its variation with annual wind statistics must be well known. The shaft torque must be known so the shaft can be built with adequate strength and the turbine load properly sized. We shall examine such performance characteristics in this chapter.

1 POWER OUTPUT FROM AN IDEAL TURBINE

The kinetic energy in a parcel of air of mass m , flowing at speed u in the x direction is

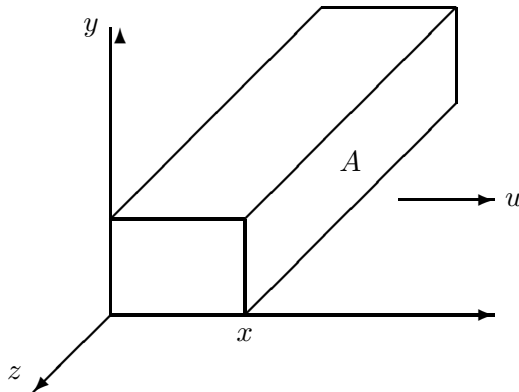
$$U = \frac{1}{2}mu^2 = \frac{1}{2}(\rho Ax)u^2 \quad \text{Joules} \quad (1)$$

where A is the cross-sectional area in m^2 , ρ is the air density in kg/m^3 , and x is the thickness of the parcel in m. If we visualize the parcel as in Fig. 1 with side x moving with speed u and the opposite side fixed at the origin, we see the kinetic energy increasing uniformly with x , because the mass is increasing uniformly.

The power in the wind, P_w , is the time derivative of the kinetic energy:

$$P_w = \frac{dU}{dt} = \frac{1}{2}\rho Au^2 \frac{dx}{dt} = \frac{1}{2}\rho Au^3 \quad \text{W} \quad (2)$$

This can be viewed as the power being supplied at the origin to cause the energy of the parcel to increase according to Eq. 1. A wind turbine will extract power from side x , with Eq. 2 representing the total power available at this surface for possible extraction.

Figure 1: Packet of air moving with speed u

An expression for air density is given in Chapter 2 and is repeated here for convenience:

$$\rho = 3.485 \frac{p}{T} \quad \text{kg/m}^3 \quad (3)$$

In this equation, p is the pressure in kPa and T is the temperature in kelvin. The power in the wind is then

$$P_w = \frac{1}{2} \rho A u^3 = \frac{1.742 p A u^3}{T} \quad \text{W} \quad (4)$$

where A is area in square meters and u is wind speed in meters per second. For air at standard conditions, 101.3 kPa and 273 K, this reduces to

$$P_w = 0.647 A u^3 \quad \text{W} \quad (5)$$

The more general Eq. 4 should be used whenever the wind turbine elevation is more than a few hundred meters above sea level or the temperature is significantly above 0°C .

At standard conditions, the power in 1 m^2 of wind with a speed of 5 m/s is $0.647(5)^3 = 81 \text{ W}$. The power in the same 1 m^2 of area when the wind speed is 10 m/s is 647 W . This illustrates two basic features of wind power. One is that wind power is rather diffuse. It requires a substantial area of wind turbine to capture a significant amount of power. The other feature is that wind power varies rapidly with wind speed. Overspeed protection devices are therefore required to protect both the turbine and the load at high wind speeds.

The physical presence of a wind turbine in a large moving air mass modifies the local air speed and pressure as shown in Fig. 2. The picture is drawn for a conventional horizontal axis propeller type turbine.

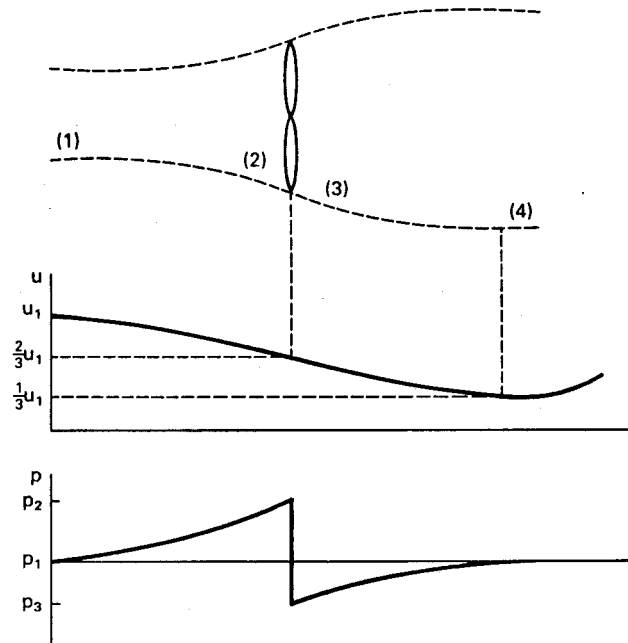


Figure 2: Circular tube of air flowing through ideal wind turbine.

Consider a tube of moving air with initial or undisturbed diameter d_1 , speed u_1 , and pressure p_1 as it approaches the turbine. The speed of the air decreases as the turbine is approached, causing the tube of air to enlarge to the turbine diameter d_2 . The air pressure will rise to a maximum just in front of the turbine and will drop below atmospheric pressure behind the turbine. Part of the kinetic energy in the air is converted to potential energy in order to produce this increase in pressure. Still more kinetic energy will be converted to potential energy after the turbine, in order to raise the air pressure back to atmospheric. This causes the wind speed to continue to decrease until the pressure is in equilibrium. Once the low point of wind speed is reached, the speed of the tube of air will increase back to $u_4 = u_1$ as it receives kinetic energy from the surrounding air[3].

It can be shown[2] that under optimum conditions, when maximum power is being transferred from the tube of air to the turbine, the following relationships hold:

$$\begin{aligned}
 u_2 &= u_3 = \frac{2}{3}u_1 \\
 u_4 &= \frac{1}{3}u_1 \\
 A_2 &= A_3 = \frac{3}{2}A_1 \\
 A_4 &= 3A_1
 \end{aligned} \tag{6}$$

The mechanical power extracted is then the difference between the input and output power in the wind:

$$P_{m,\text{ideal}} = P_1 - P_4 = \frac{1}{2}\rho(A_1u_1^3 - A_4u_4^3) = \frac{1}{2}\rho\left(\frac{8}{9}A_1u_1^3\right) \quad \text{W} \quad (7)$$

This states that 8/9 of the power in the original tube of air is extracted by an *ideal turbine*. This tube is smaller than the turbine, however, and this can lead to confusing results. The normal method of expressing this extracted power is in terms of the undisturbed wind speed u_1 and the turbine area A_2 . This method yields

$$P_{m,\text{ideal}} = \frac{1}{2}\rho\left[\frac{8}{9}\left(\frac{2}{3}A_2\right)u_1^3\right] = \frac{1}{2}\rho\left(\frac{16}{27}A_2u_1^3\right) \quad \text{W} \quad (8)$$

The factor $16/27 = 0.593$ is sometimes called the *Betz coefficient*. It shows that an actual turbine cannot extract more than 59.3 percent of the power in an undisturbed tube of air of the same area. In practice, the fraction of power extracted will always be less because of mechanical imperfections. A good fraction is 35-40 percent of the power in the wind under optimum conditions, although fractions as high as 50 percent have been claimed. A turbine which extracts 40 percent of the power in the wind is extracting about two-thirds of the amount that would be extracted by an ideal turbine. This is rather good, considering the aerodynamic problems of constantly changing wind speed and direction as well as the frictional loss due to blade surface roughness.

It is interesting to note that the total pressure difference across the turbine is rather small. For a 6 m/s wind speed, p_2 will be about 12.6 Pa greater than p_1 , while p_3 will be about 7.6 Pa less. The pressure difference is then about 0.02 percent of the ambient pressure. Small pressure differences are therefore able to provide rather substantial turbine power outputs.

2 AERODYNAMICS

Air flow over a stationary airfoil produces two forces, a *lift* force perpendicular to the air flow and a *drag* force in the direction of air flow, as shown in Fig. 3. The existence of the lift force depends upon *laminar* flow over the airfoil, which means that the air flows smoothly over both sides of the airfoil. If turbulent flow exists rather than laminar flow, there will be little or no lift force. The air flowing over the top of the airfoil has to speed up because of a greater distance to travel, and this increase in speed causes a slight decrease in pressure. This pressure difference across the airfoil yields the lift force, which is perpendicular to the direction of air flow.

The air moving over the airfoil also produces a drag force in the direction of the air flow. This is a loss term and is minimized as much as possible in high performance wind turbines.

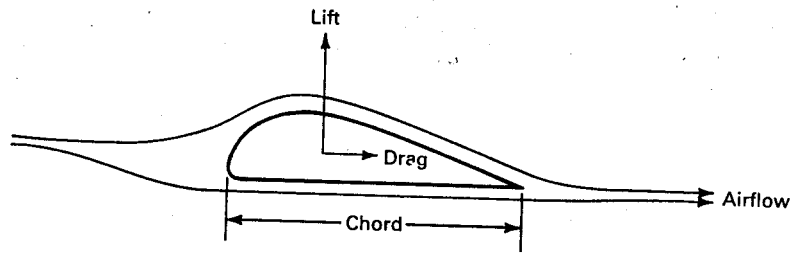


Figure 3: Lift and drag on a stationary airfoil.

Both the lift and the drag are proportional to the air density, the area of the airfoil, and the square of the wind speed.

Suppose now that we allow the airfoil to move in the direction of the lift force. This motion or translation will combine with the motion of the air to produce a relative wind direction shown in Fig. 4. The airfoil has been reoriented to maintain a good lift to drag ratio. The lift is perpendicular to the relative wind but is not in the direction of airfoil translation.

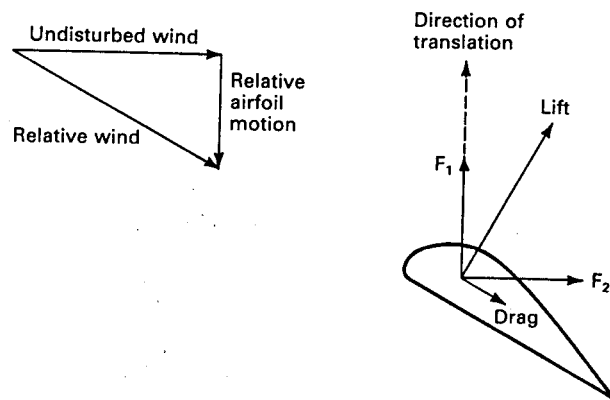


Figure 4: Lift and drag on a translating airfoil.

The lift and drag forces can be split into components parallel and perpendicular to the direction of the undisturbed wind, and these components combined to form the net force F_1 in the direction of translation and the net force F_2 in the direction of the undisturbed wind. The force F_1 is available to do useful work. The force F_2 must be used in the design of the airfoil supports to assure structural integrity.

A practical way of using F_1 is to connect two such airfoils or blades to a central hub and allow them to rotate around a horizontal axis, as shown in Fig. 5. The force F_1 causes a torque which drives some load connected to the propeller. The tower must be strong enough to withstand the force F_2 .

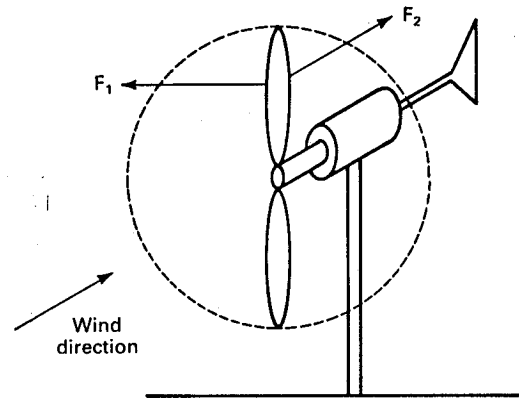
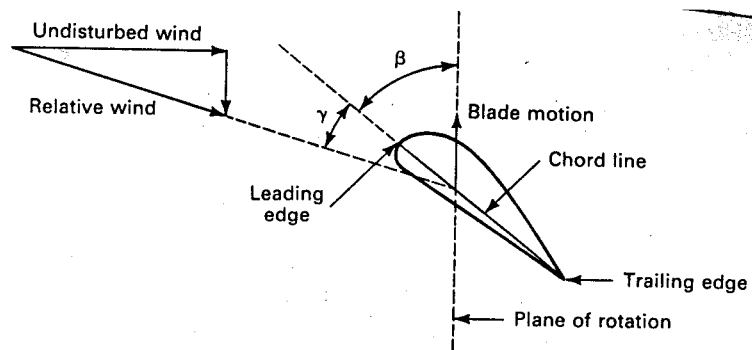


Figure 5: Aerodynamic forces on a turbine blade.

These forces and the overall performance of a wind turbine depend on the construction and orientation of the blades. One important parameter of a blade is the *pitch angle*, which is the angle between the chord line of the blade and the plane of rotation, as shown in Fig. 6. The *chord line* is the straight line connecting the leading and trailing edges of an airfoil. The *plane of rotation* is the plane in which the blade tips lie as they rotate. The blade tips actually trace out a circle which lies on the plane of rotation. Full power output would normally be obtained when the wind direction is perpendicular to the plane of rotation. The pitch angle is a static angle, depending only on the orientation of the blade.

Figure 6: Definition of pitch angle β and angle of attack γ .

Another important blade parameter is the *angle of attack*, which is the angle γ between the chord line of the blade and the relative wind or the effective direction of air flow. It is a dynamic angle, depending on both the speed of the blade and the speed of the wind. The blade speed at a distance r from the hub and an angular velocity ω_m is $r\omega_m$.

A blade with twist will have a variation in angle of attack from hub to tip because of the

variation of $r\omega_m$ with distance from the hub. The lift and drag have optimum values for a single angle of attack so a blade without twist is less efficient than a blade with the proper twist to maintain a nearly constant angle of attack from hub to tip. Even the blades of the old Dutch windmills were twisted to improve the efficiency. Most modern blades are twisted, but some are not for cost reasons. A straight blade is easier and cheaper to build and the cost reduction may more than offset the loss in performance.

When the blade is twisted, the pitch angle will change from hub to tip. In this situation, the pitch angle measured three fourths of the distance out from the hub is selected as the reference.

3 POWER OUTPUT FROM PRACTICAL TURBINES

The fraction of power extracted from the power in the wind by a practical wind turbine is usually given the symbol C_p , standing for the coefficient of performance. Using this notation and dropping the subscripts of Eq. 8 the actual mechanical power output can be written as

$$P_m = C_p \left(\frac{1}{2} \rho A u^3 \right) = C_p P_w \quad \text{W} \quad (9)$$

The coefficient of performance is not a constant, but varies with the wind speed, the rotational speed of the turbine, and turbine blade parameters like angle of attack and pitch angle.

The Darrieus turbines operate with fixed pitch while the large horizontal axis turbines normally have variable pitch. The pitch is varied to hold C_p at its largest possible value up to the rated speed u_R of the turbine, and then is varied to reduce C_p while P_w continues to increase with wind speed, in order to maintain the output power at its rated value, P_{mR} . This is shown in Fig. 7.

It is not practical to hold C_p constant with pitch control because of manufacturing and control limitations, so it will vary with wind speed even for a fixed rotational speed, variable pitch blade. A variation of C_p versus u is shown in Fig. 8 for the MOD-2 wind turbine[1, 8]. The turbine starts producing power at a hub height wind speed of 6.3 m/s (14 mi/h) and a C_p of about 0.28. A maximum C_p of 0.41, defined as C_{pm} , occurs at 9 m/s (20 mi/h). Designing the blades to have a maximum coefficient of performance below the rated wind speed helps to maximize the energy production of the turbine.

The rated wind speed for the MOD-2 is 12.3 m/s (27.5 mi/h) at hub height. C_p has dropped to about 0.36 at this wind speed. The coefficient of performance at rated wind speed can be defined as C_{pR} . Two curves for C_p are shown in Fig. 8 for wind speeds above the rated wind speed, the upper curve showing the capability of the rotor and the lower curve showing C_p under actual operating conditions. The turbine is shut down at 20 m/s (45 mi/h) to prevent damage from such high winds, and the actual C_p is well under 0.1 when this wind

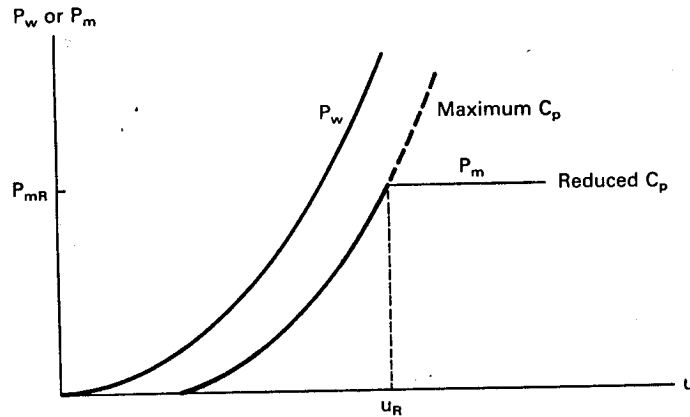


Figure 7: Shaft power output of a variable-pitch turbine.

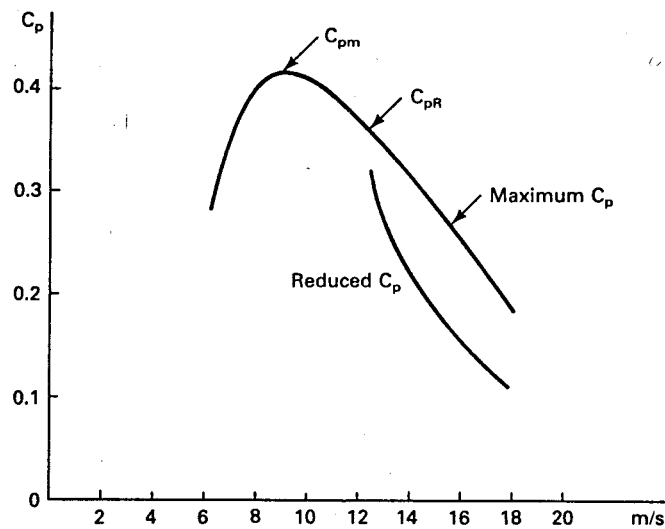


Figure 8: Coefficient of performance versus wind speed for MOD-2.

speed is reached.

The curve shown in Fig. 8 is only valid for one rotational speed, in this case 17.5 revolutions per minute (r/min). When the rotational speed is changed, $r\omega_m$ changes and causes the angle of attack to change. This in turn changes C_p at a given wind speed. It is often convenient for design purposes to have a single curve for C_p , from which the effects of changing either rotational speed or wind speed can be determined. This means that the rotational speed and the wind speed must somehow be combined into a single variable before such a single curve can be drawn. Experiments show that this single variable is the ratio of the turbine *tip speed*

$r_m\omega_m$ to the wind speed u . This *tip speed ratio* is defined as

$$\lambda = \frac{r_m\omega_m}{u} \quad (10)$$

where r_m is the maximum radius of the rotating turbine in m, ω_m is the *mechanical angular velocity* of the turbine in rad/s, and u is the undisturbed wind speed in m/s.

The angular velocity ω_m is determined from the *rotational speed* n (r/min) by the equation

$$\omega_m = \frac{2\pi n}{60} \quad \text{rad/s} \quad (11)$$

The variation of C_p with λ for the Sandia 17-m Darrieus[10] is shown in Fig. 9. This particular machine will be used for illustration purposes in this chapter. All horizontal axis propeller turbines and other Darrieus machines will have generally similar curves. This curve is for a machine similar to the one shown in Fig. 1.5 with the difference being that each blade has two struts extending from the blade to the center of the vertical shaft. Performance is somewhat better without the aerodynamic losses introduced by the struts, but this will not affect our discussion. This particular machine has a rotor diameter of 16.7 m, a rotor height of 17 m, and a rotor swept area of 187 m².

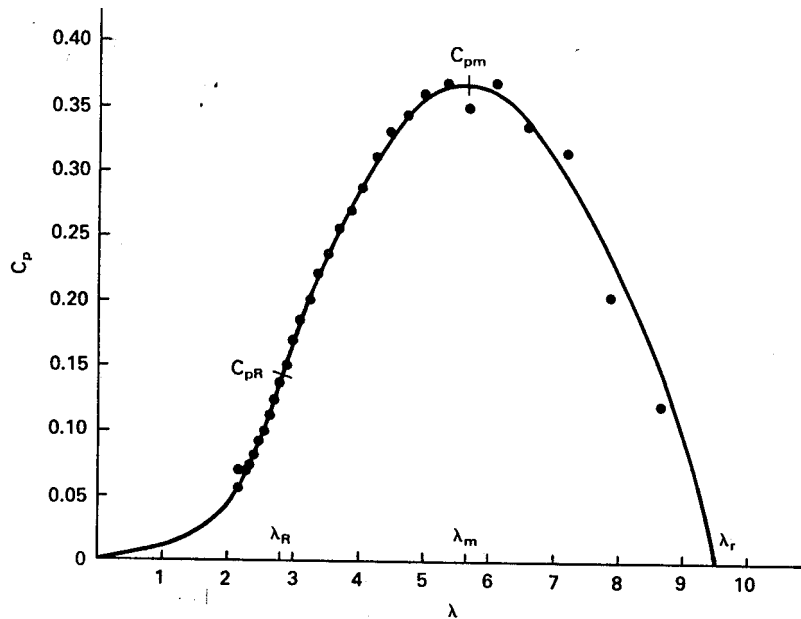


Figure 9: Coefficient of performance C_p versus tip-speed ratio λ for Sandia 17-m Darrieus turbine. Two blades; 42 r/min.

The size of this machine was chosen on the basis of available aluminum forming equipment

and other hardware and may not be an optimum size. A good design procedure is to select several sizes, perhaps arbitrarily, and then determine manufacturing costs, energy production, and unit energy costs for each size. The detailed designs for each size reveal possible difficulties that do not appear in the conceptual design stage. The fact that no one builds a satisfactory gearbox, or that no one can extrude the aluminum blade in the desired size, would probably not be discovered until the detailed design stage. Such designs, with certain sizes and operating conditions arbitrarily selected to allow a detailed design to continue, are often called *point designs*. Other Darrieus point designs will be mentioned later in the chapter.

The Darrieus is operated as a fixed pitch turbine since there is no convenient way of varying the pitch. The blade motion causes the relative direction of the air seen by the blade to change continuously during a revolution. This means that the angle of attack is continuously changing during rotation. The coefficient of performance shown in Fig. 9 is therefore an average value for one complete revolution.

As mentioned earlier, the Darrieus turbine is normally not self starting. Fig. 9 indicates that C_p is very low for a Darrieus turbine operating at tip speed ratios below about two. The correspondingly low shaft power is insufficient to overcome friction so the Darrieus turbine needs a mechanical assist to get its tip speed up to at least twice the wind speed. At tip speed ratios above two, the Darrieus is able to extract enough power from the wind to accelerate itself up to the desired operating angular velocity. As it accelerates, it passes through the rated coefficient of performance C_{pR} at λ_R , reaching the maximum coefficient of performance C_{pm} at λ_m . If there is no load on the turbine it will continue to accelerate until the *runaway* tip speed ratio λ_r is reached. In high winds, the turbine angular velocity may easily exceed design limits at λ_r , hence the turbine should not be operated without a load.

The normal operating mode of a large wind turbine will have the turbine rotating at fixed rotational speed (e.g. 42 r/min for the data of Fig. 9). For fixed $r_m\omega_m$, the tip speed ratio will be large for a low wind speed and get smaller as the wind speed increases. As the wind speed increases from a small value the mechanical power output increases due to both the greater power in the wind and the larger values of C_p . This variation is shown in Fig. 10. Eventually C_p reaches its maximum C_{pm} at the tip speed ratio λ_m . For higher wind speeds (lower tip speed ratios) the power in the wind continues to increase while C_p starts to decrease. The product of $C_p P_w$ continues to increase until the rated mechanical power output P_{mR} is reached at C_{pR} and λ_R . After that point, C_p decreases at an even greater rate than before, so P_m starts a slow decrease.

It should be mentioned that the rated power can be selected at a value below the maximum possible power. In fact, this may be a common practice for purposes of guarantees. A Darrieus turbine which can produce 30 kW at a wind speed of 12 m/s may be rated at 25 kW at a wind speed of 10 m/s, for example. A customer testing a machine would find power flows equal to and slightly above the nameplate rating and would conclude that the machine was performing as advertised. If the actual power never reached the advertised rating, due perhaps to manufacturing tolerances or installation errors, the customer may become angry and initiate legal action against the manufacturer.

We could therefore distinguish between the coefficient of performance and tip speed ratio at rated wind speed and at the wind speed where maximum power is actually obtained. This may be necessary in some situations, but normally is not required. We shall therefore use C_{pR} and λ_R to refer to both the rated power and the maximum power cases.

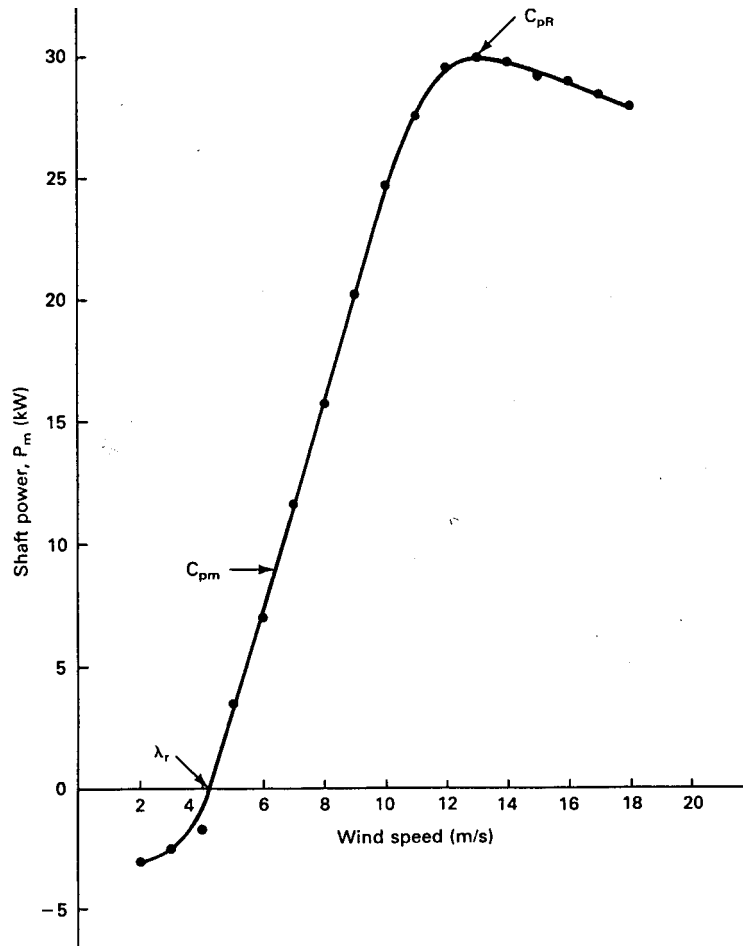


Figure 10: Shaft power output of Sandia 17-m Darrieus turbine at 42 r/min in an average pressure of 83 kPa and an average temperature of 15°C.

Figure 10 shows experimental data recorded at an average temperature of 15°C and an average pressure of 83 kPa. The power output of the same turbine in air at standard conditions would be nearly 30 percent greater at the same wind speeds.

The lack of a convenient means for changing the pitch of the Darrieus blades is seen to not be a serious disadvantage because of this self limiting characteristic of power output. If the fixed speed load is able to accept the maximum possible mechanical power, no additional braking or loading is necessary as the wind speed increases above its rated value.

Let us now consider the effect of changing the rated rotational speed on the operation of the turbine. A higher rotational speed means that a given value of λ will occur at a higher wind speed. If the turbine characteristic does not change with rotational speed, then the same C_{pR} applies at the same λ_R , which is at a higher wind speed than before. The higher wind speed means that a larger shaft power will be delivered. A 25 percent increase in wind speed means the power in the wind has increased by $(1.25)^3 = 1.95$. Therefore, if we operate the turbine of Figs. 9 and 10 at a 25 percent higher rotational speed [$42(1.25) = 52.5$ r/min], we would expect approximately twice the peak shaft power output observed at 42 r/min. This indeed is the case, as shown by Fig. 11. In fact, C_{pR} has increased slightly so our 30-kW (mechanical power) machine at 42 r/min has become a 67-kW machine at 52.5 r/min.

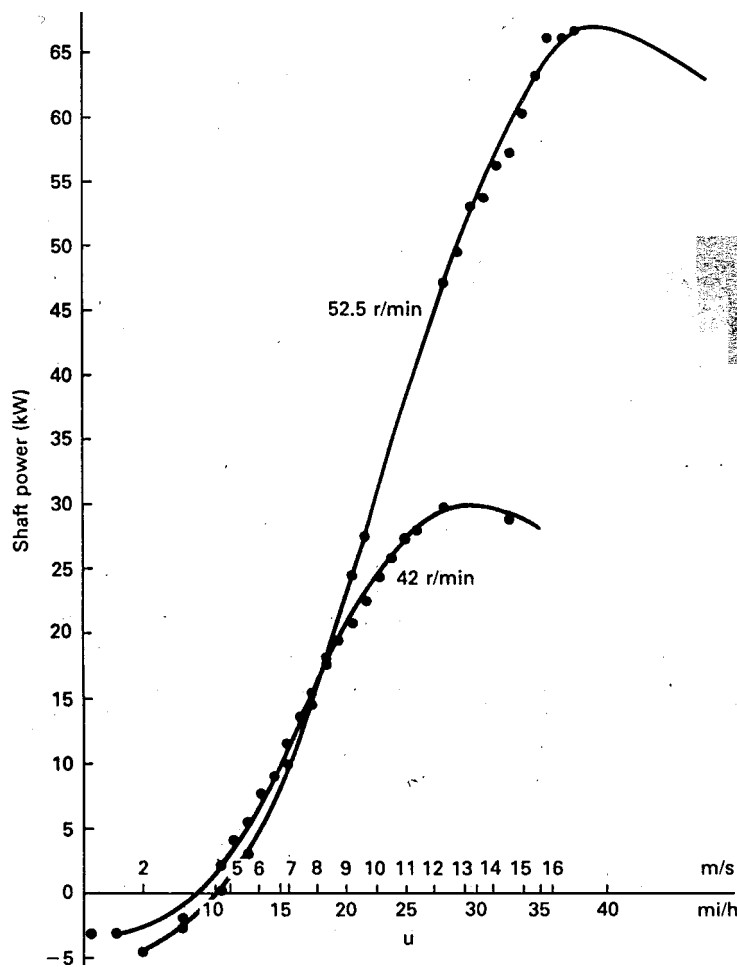


Figure 11: Shaft power output of Sandia 17-m Darrieus at two angular velocities in an ambient pressure of 83 kPa and an ambient temperature of 15°C.

At first glance, it would appear that 52.5 r/min would be a superior choice over 42 r/min. This may not be the case, however, because the extra power is available only at the higher

wind speeds, above about 9 m/s. Below 9 m/s the power output at 52.5 r/min is actually less than for 42 r/min. Wind speeds below 9 m/s are usually more common than speeds above 9 m/s, so additional power output at higher wind speeds may be more than offset by reduced power output at lower wind speeds. The choice of rated rotational speed therefore depends on the wind regime of a given site. A site with a mean wind speed of 9 m/s could probably justify the 52.5-r/min machine while a site with a mean speed of 6 m/s could not. We shall consider a more detailed analysis of this choice later in the chapter.

4 TRANSMISSION AND GENERATOR EFFICIENCIES

The shaft power output that we have been discussing is not normally used directly, but is usually coupled to a load through a transmission or gear box. The load may be a pump, compressor, grinder, electrical generator, and so on. For purposes of illustration, we will consider the load to be an electrical generator. The basic system is then as shown in Fig. 12. We start with the power in the wind, P_w . After this power passes through the turbine, we have a mechanical power P_m at the turbine angular velocity ω_m , which is then supplied to the transmission. The *transmission output power* P_t is given by the product of the turbine output power P_m and the *transmission efficiency* η_m :

$$P_t = \eta_m P_m \quad \text{W} \quad (12)$$

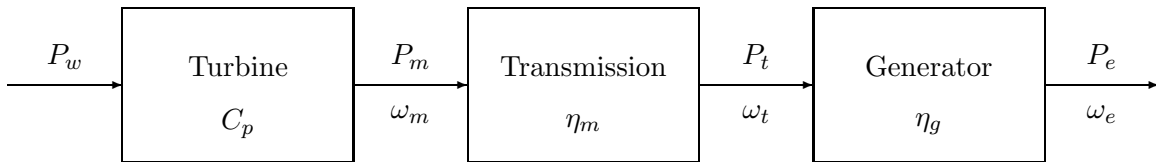


Figure 12: Wind electric system

Similarly, the generator output power P_e is given by the product of the transmission output power and the generator efficiency η_g :

$$P_e = \eta_g P_t \quad \text{W} \quad (13)$$

Equations 9, 12, and 13 can be condensed to a single equation relating electrical power output to wind power input:

$$P_e = C_p \eta_m \eta_g P_w \quad \text{W} \quad (14)$$

At rated wind speed, the rated electrical power output can be expressed as

$$P_{eR} = C_{pR}\eta_{mR}\eta_{gR}\frac{\rho}{2}Au_R^3 \quad \text{W} \quad (15)$$

where C_{pR} is the coefficient of performance at the rated wind speed u_R , η_{mR} is the transmission efficiency at rated power, η_{gR} is the generator efficiency at rated power, ρ is the air density, and A is the turbine area.

The quantity $C_{pR}\eta_{mR}\eta_{gR}$ is the *rated overall efficiency* of the turbine. We shall give this quantity a symbol of its own, η_o :

$$\eta_o = C_{pR}\eta_{mR}\eta_{gR} \quad (16)$$

It should be mentioned that the American Wind Energy Association is trying to avoid the use of the term *rated power* in favor of *maximum power*. Many wind turbine distributors will also refuse to use the term *rated power*. The reason for this is the tendency for the uninformed to attach more significance to this quantity than it deserves. With conventional generators, a 60-kW generator priced at \$1000 per kilowatt is almost always a better buy than a 25-kW generator priced at \$2000 per kilowatt. This does not necessarily hold true for wind turbines since the 60-kW and the 25-kW wind turbine systems may be the same turbine with a larger transmission and synchronous generator in the 60-kW version. One salesman is asking \$60,000 for almost the same machine being offered by someone else for \$50,000. The higher price is being disguised by quoting the price in dollars per unit rating. This situation can lead to much confusion as well as some unethical behavior.

With this cautionary note, we shall retain the use of the term *rated power*, but we shall try not to give it more significance than it deserves. We shall restrict its use to theoretical models where the rated power occurs at the rated wind speed u_R at a sharp corner of the power output versus wind speed curve. We shall then use the *maximum power* to refer to the peak value seen on the experimental power output curve.

We shall see a better way of describing the performance of a given wind turbine in the next section. Rather than either rated or maximum power, it is the energy production that one could expect from a given turbine in a given wind regime.

Example

The Sandia 17-m Darrieus is rated at 60 kW at 15.5 m/s and 52.5 r/min, and at 25 kW at 11 m/s and 42 r/min. The area A is 187 m². Compute the rated overall efficiency at each rating and standard conditions.

At standard conditions, $\rho/2 = 0.647$. Inserting this value in Eqs. 15 and 16 we get

$$\eta_{0,25} = \frac{25,000}{0.647(187)(11)^3} = 0.155$$

$$\eta_{0,60} = \frac{60,000}{0.647(187)(15.5)^3} = 0.133$$

These results illustrate the fact that the rated overall efficiency may be significantly lower than the maximum coefficient of performance of the turbine itself. This is not a major problem if the various efficiencies are high below the rated wind speed. For wind speeds at or above rated, the power in the wind is large enough that somewhat lower efficiencies do not prevent rated power from being reached.

The rated overall efficiency just defined is only valid at rated wind speed. We need to know the overall efficiency at lower wind speeds to determine the energy production of the turbine, so we need to determine the individual efficiencies. We have already examined the variation of C_p , so we shall now consider η_m and η_g .

Transmission losses are primarily due to viscous friction of the gears and bearings turning in oil. At fixed rotational speed, the losses do not vary strongly with transmitted torque. It is therefore reasonable to assume that the transmission loss is a fixed percentage of the low speed shaft rated power. The actual percentage will vary with the quality of the transmission, but a reasonable value seems to be 2 percent of rated power per stage of gears. The maximum practical gear ratio per stage is approximately 6:1, so two or three stages of gears are typically required. Two stages would have a maximum allowable gear ratio of $(6)^2:1 = 36:1$ so any design requiring a larger gear ratio than this would use three stages.

Suppose that q is the number of gear stages. The transmission efficiency is then

$$\eta_m = \frac{P_t}{P_m} = \frac{P_m - (0.02)qP_{mR}}{P_m} \quad (17)$$

where P_{mR} is the rated turbine shaft power.

This equation is plotted in Fig. 13 for one, two, and three stages. It can be seen that the transmission efficiency is not very good for low power inputs. It is therefore desirable to choose ratings such that the transmission is operating above the knee of the curve in Fig. 13 as much as possible.

Example

How many gear stages are required in the transmission for the Sandia 17-m Darrieus to drive a 1800 r/min generator for each of the proposed speeds of 42 and 52.5 r/min? Assume the maximum gear ratio for a single stage is 6:1.

The overall gear ratio at 42 r/min is $1800/42 = 42.86:1$, while at 52.5 r/min it is $1800/52.5 = 34.29:1$. Operating at 42 r/min requires a 3 stage transmission while a 2 stage transmission would be adequate at 52.5 r/min. The transmission for the 52.5 r/min system will therefore be more efficient and probably less expensive than the corresponding transmission for the 42 r/min system. This would encourage us to use the higher speed system, if possible.

It should be mentioned that synchronous generators are also made to operate at 1200 r/min for only a small increase in cost over the 1800 r/min version. Therefore, the possibility of using a 1200 r/min generator should be examined if the 42 r/min mode is selected. This would present an overall gear ratio of $1200/42 = 28.57:1$, which could be accomplished with a two stage transmission.

The generator losses may be considered in three categories: hysteresis and eddy current

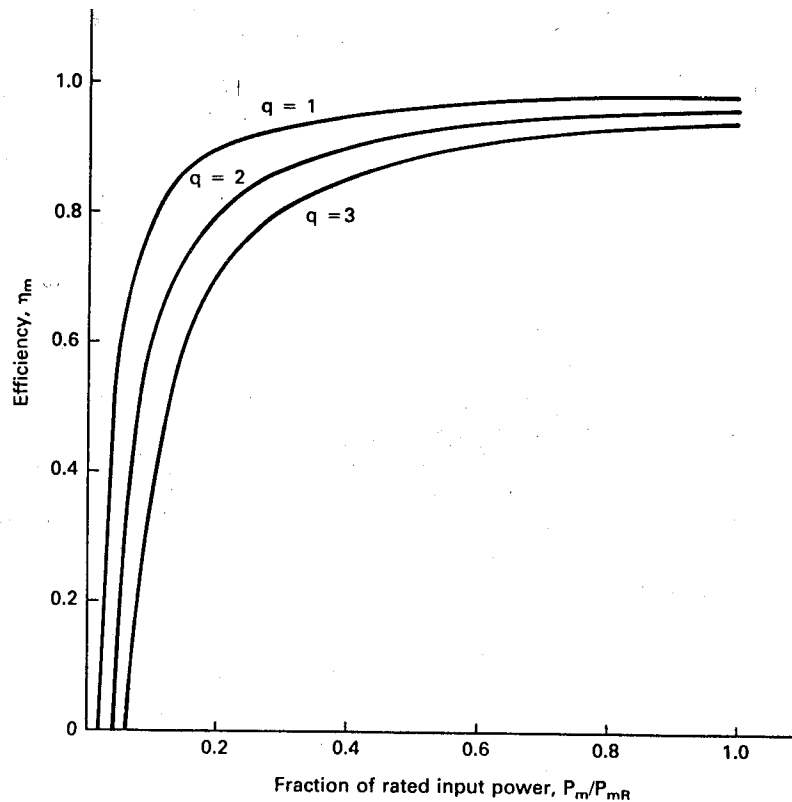


Figure 13: Transmission efficiency for one, two, and three stages, with 2 percent loss per stage.

losses, which are functions of the operating voltage and frequency, windage and bearing friction losses, which vary with rotational speed, and copper losses, which vary as the square of the load or output current. Normal operation with the generator connected to the utility grid will be with fixed voltage and frequency, and either fixed or almost fixed angular velocity depending on whether the generator is of the synchronous or induction type. These generators will be discussed in more detail in the next chapter.

It is appropriate to group the losses into two categories: fixed and variable, with hysteresis, eddy currents, windage, and bearing friction considered fixed, and copper losses being variable. The relative magnitudes of these losses will vary with the design of the generator. It is considered good design to have the two categories approximately equal to each other when the generator is delivering rated power, and we will assume this for discussion purposes.

Larger generators are inherently more efficient than smaller generators. Some losses are proportional to the surface area of the rotor while the rated electrical power is proportional to the volume. The ratio of volume to area increases with increased physical size, hence the efficiency goes up. Good quality generators may have full load efficiencies of 0.85 for a 2-kW

rating, 0.9 for a 20-kW rating, 0.93 for a 200-kW rating, and 0.96 for a 2-MW rating. The efficiency continues to climb with size, exceeding 0.98 for the very large generators in coal and nuclear power plants. This variation in efficiency with rating is different from the efficiencies of the turbine and transmission, which were assumed to not vary with size. The differences between very small and large generators are significant, and should be included in any detailed economic study.

The effects of rated power and actual power on generator efficiency can all be combined in an empirical equation[10]. When expressed in terms of the input shaft power to the generator, this expression is

$$\eta_g = \frac{X - (0.5)Y(1 - Y)(X^2 + 1)}{X} \quad (18)$$

where the parameters X and Y are given by

$$X = \frac{P_t}{P_{tR}} \quad (19)$$

$$Y = 0.05 \left(\frac{10^6}{P_{eR}} \right)^{0.215} \quad (20)$$

In these equations, P_{tR} and P_{eR} are the *rated mechanical power input* and the *rated electrical power output* in watts of the generator. Equation eq:4.18 is plotted in Fig. 14 for three rated generator sizes: 20 kW (20×10^3 W), 200 kW, and 2000 kW. The curves are seen to be very similar in shape to the transmission efficiency curves of Fig. 13.

The power output of the electrical generator can now be determined, conceptually at least, by finding C_p , η_m and η_g for a given turbine and wind speed, multiplying them together to find the overall efficiency, and then multiplying that by the power in the wind. This can be done by reading values from graphs or by analytical techniques if the appropriate mathematical models have been defined. Design values of turbine rated rotational speed and rated sizes of the transmission and generator can be varied, and the process repeated. Optimum values can be determined which will maximize the energy production per dollar of investment.

The selection of ratings is somewhat of an art, partly because commercial products are made in discrete size increments. A company which manufactured a 25-kW and a 30-kW generator would probably not manufacture a 27-kW generator. We are therefore forced to choose a size which is not exactly equal to the theoretically desired value. Consider, for example, the Sandia 17-m Darrieus turbine with shaft power production shown in Fig. 10. The peak shaft power is 30 kW. We would want to select a transmission of at least this input rating. If there is a 30-kW transmission built for this class of service, it would be selected. Otherwise, a 35- or 40-kW transmission would probably be chosen. This would allow a safety factor and perhaps increase the operational life of the transmission.

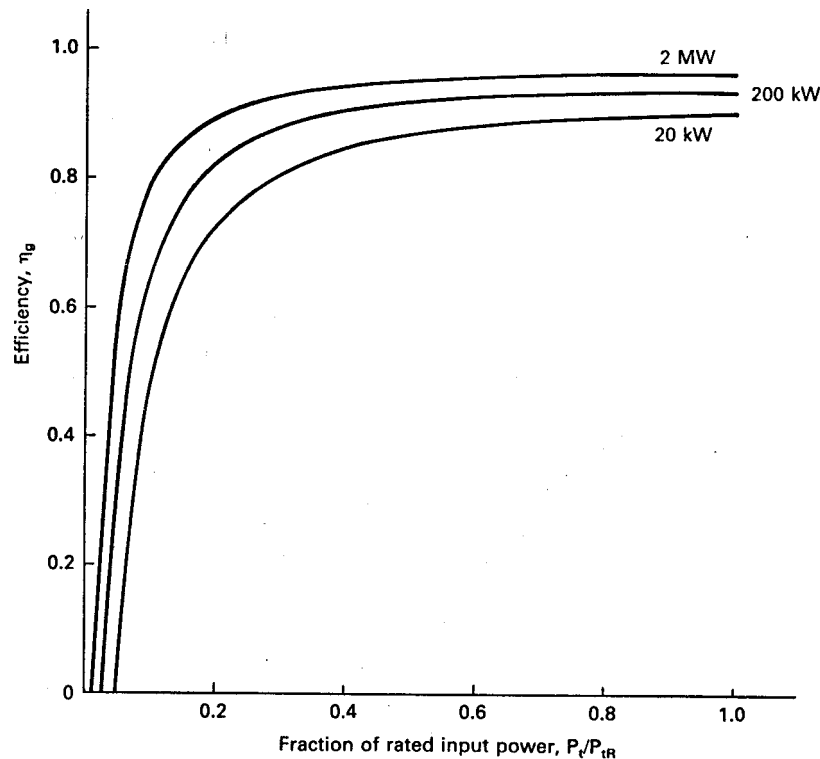


Figure 14: Generator efficiency for three generator sizes.

If a two stage, 30-kW transmission with an efficiency curve such as Fig. 13 is selected, the rated output power is $(0.96)(30) = 28.8$ kW. Generators are always rated in terms of output power, so a 25-kW generator with efficiency of 0.9 has a rated input power $25/0.9 = 27.8$ kW. The next size up, say 30 kW, would have a rated input power of $30/0.9 = 33.3$ kW. Should we select the generator that is slightly undersized, or should we choose the next larger unit? In this particular situation, a good case can be made for choosing the smaller generator. There will be some slight savings in cost and weight, and some increase in average system efficiency because the generator will be operated at a higher fraction of its rating. The slight overload is acceptable because it is not present all the time. Any generator can supply 10 or 20 percent greater power than its rating for periods up to an hour if it is allowed to cool after that period. The wind is variable enough that periods of slight overload will be compensated by other periods of lighter load, so the average power delivered in a period of perhaps one hour would not be above the rated power.

It should be remembered that the heat conduction away from the generator is greater in higher winds, and that the generator rating is determined for indoor or calm conditions. This effect may increase the practical rating of the generator by 5 percent or so. These factors of variable power operation and increased air cooling make it permissible to size the generator

by as much as 10 percent under the predicted steady state requirement.

If we choose the 25-kW generator and connect it to the turbine whose shaft power is shown in Fig. 10, and if a two-stage transmission is assumed, the electrical power output as a function of wind speed will be as shown in Fig. 15. The shaft power input is also shown for comparison purposes. It is seen that both the shaft power and the electrical power output increase nearly linearly with wind speed up to their maximum values. This may seem somewhat surprising since the power in the wind increases as the cube of the wind speed. It is correct, however, since the low efficiencies at low wind speeds are responsible for linearizing the power output curve.

We note in Fig. 15 that the electrical power output rises above zero at a wind speed of about 5 m/s. This wind speed at which electrical power production starts is called the *cut-in speed* u_c . The turbine will develop enough mechanical power to rotate itself at slightly lower speeds, but this wind speed will actually supply all the generator and transmission losses so useful electrical power can be produced.

Fig. 15 has been developed from actual turbine data and from reasonably complete models of the transmission and generator. Other turbines, transmissions, and generators will produce somewhat different curves with approximately the same shape.

It is convenient to define a model for P_e that can be used in discussing any wind system. The simplest model would use a straight line to describe the variation in output power between cut-in and rated wind speeds. A straight line describes the output of the Sandia 17-m Darrieus rather well. We must remember, of course, that other monotonic functions will fit the observed data nearly as good as a straight line, or perhaps even better for some machines, and may yield more accurate energy estimates or more convenient analytic results. It will be seen later that a closed form expression for energy production can be obtained if P_e is assumed to vary as u^k between cut-in and rated wind speeds, where k is the Weibull shape parameter. Numerical integration is required if P_e is assumed to vary as u , or in a linear fashion. Therefore, our choice of a somewhat complicated model will make later computations easier, and perhaps more accurate, than the choice of the simplest possible model. We therefore define the following equations for the electrical power output of a model wind turbine[9]:

$$\begin{aligned}
 P_e &= 0 && (u < u_c) \\
 P_e &= a + bu^k && (u_c \leq u \leq u_R) \\
 P_e &= P_{eR} && (u_R < u \leq u_F) \\
 P_e &= 0 && (u > u_F)
 \end{aligned} \tag{21}$$

In the expression, P_{eR} is the *rated electrical power*, u_c is the *cut-in* wind speed, u_R is the *rated* wind speed, u_F is the *furling* wind speed, and k is the Weibull shape parameter. Furling

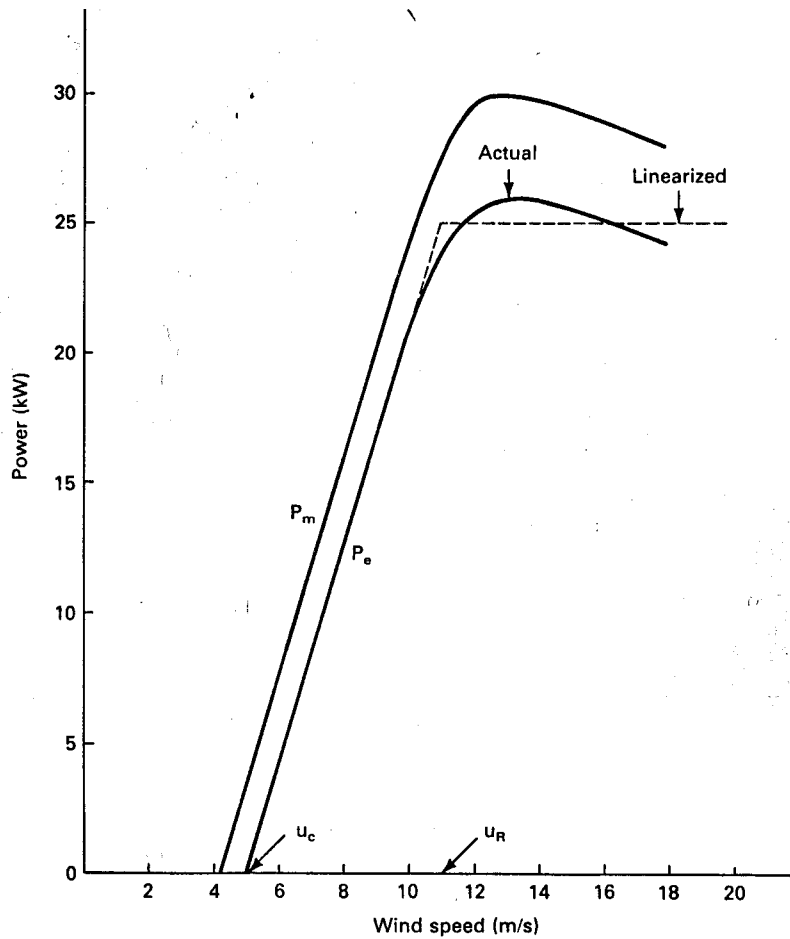


Figure 15: Electrical power output compared with shaft power of Sandia 17-m Darrieus turbine operating at 42 r/min.

is an old sailing term which refers to the process of rolling up the canvas sails in anticipation of high winds. It therefore is used to refer to the wind speed at which the turbine is shut down to prevent structural damage. This condition normally occurs only a few hours during the year, and therefore does not have a large influence on energy production.

The coefficients a and b are given by

$$a = \frac{P_{eR} u_c^k}{u_c^k - u_R^k}$$

$$b = \frac{P_{eR}}{u_R^k - u_c^k} \quad (22)$$

As mentioned in Chapter 2, the Rayleigh distribution is a special case of the Weibull distribution with $k = 2$ and is often sufficiently accurate for analysis of wind power systems. This value of k should be used if the wind statistics at a given site are not well known.

A plot of P_e versus u is shown in Fig. 16, for $k = 2$. P_e varies as u^k between the cut-in and rated wind speeds. It is then assumed to be a constant value between the rated and furling wind speeds. At the furling wind speed u_F the turbine is shut down to protect it from high winds.

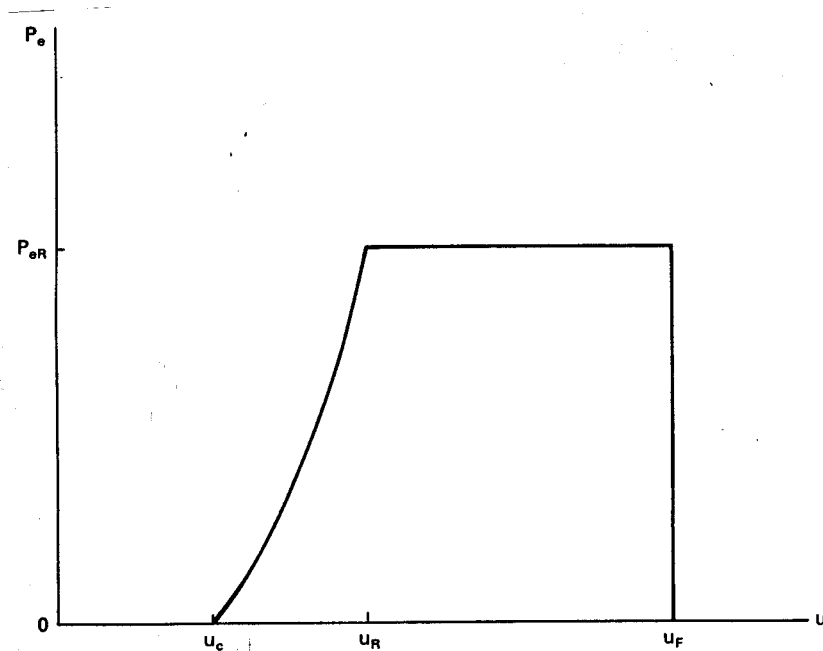


Figure 16: Model wind turbine output versus wind speed.

5 ENERGY PRODUCTION AND CAPACITY FACTOR

We have seen that the electrical power output of a wind turbine is a function of the wind speed, the turbine angular velocity, and the efficiencies of each component in the drive train. It is also a function of the type of turbine (propeller, Darrieus, etc.), the inertia of the system,

and the gustiness of the wind. We will assume that the power output can be adequately described by the model of Eqs. 21, although more sophisticated models might be necessary in rare cases. We now want to combine the variation in output power with wind speed with the variation in wind speed at a site to find the *average power* $P_{e,ave}$ that would be expected from a given turbine at a given site. The average power output of a turbine is a very important parameter of a wind energy system since it determines the total energy production and the total income. It is a much better indicator of economics than the rated power, which can easily be chosen at too large a value.

The average power output from a wind turbine is the power produced at each wind speed times the fraction of the time that wind speed is experienced, integrated over all possible wind speeds.

In integral form, this is

$$P_{e,ave} = \int_0^{\infty} P_e f(u) du \quad \text{W} \quad (23)$$

where $f(u)$ is a probability density function of wind speeds. We shall use the Weibull distribution

$$f(u) = \frac{k}{c} \left(\frac{u}{c}\right)^{k-1} \exp \left[- \left(\frac{u}{c}\right)^k \right] \quad (24)$$

as described in Chapter 2.

Substituting Eqs. 21 and 24 into Eq. 23 yields

$$P_{e,ave} = \int_{u_c}^{u_R} (a + bu^k) f(u) du + P_{eR} \int_{u_R}^{u_F} f(u) du \quad \text{W} \quad (25)$$

There are two distinct integrals in Eq. 25 which need to be integrated. One has the integrand $u^k f(u)$ and the other has the integrand $f(u)$. The integration can be accomplished best by making the change in variable

$$x = \left(\frac{u}{c}\right)^k \quad (26)$$

The differential dx is then given by

$$dx = k \left(\frac{u}{c}\right)^{k-1} d\left(\frac{u}{c}\right) \quad (27)$$

The two distinct integrals of Eq. 25 can therefore be written as

$$\int f(u)du = \int e^{-x}dx = -e^{-x} \quad (28)$$

$$\begin{aligned} \int u^k f(u)du &= \int c^k \left(\frac{u^k}{c^k} \right) f(u)du \\ &= c^k \int x e^{-x} dx = -c^k (x+1)e^{-x} \end{aligned} \quad (29)$$

When we substitute the limits of integration into Eq. 25, and reduce to the minimum number of terms, the result is

$$P_{e,ave} = P_{eR} \left\{ \frac{\exp[-(u_c/c)^k] - \exp[-(u_R/c)^k]}{(u_R/c)^k - (u_c/c)^k} - \exp \left[- \left(\frac{u_F}{c} \right)^k \right] \right\} W \quad (30)$$

We now have an equation which shows the effects of cut-in, rated, and furling speeds on the average power production of a turbine. For a given wind regime with known c and k parameters, we can select u_c , u_R , and u_F to maximize the average power, and thereby maximize the total energy production. There are relationships among u_c , u_R , and u_F which must be considered, however, if realistic results are to be expected. The wind must contain enough power at the cut-in speed to overcome all the system losses. A cut-in speed of $0.5u_R$ would imply that the gearbox and generator losses at cut-in are the fraction $(0.5)^3 = 0.125$ of rated power. A cut-in speed of $0.4u_R$ implies that the losses in that case are the fraction $(0.4)^3 = 0.064$ of rated power. It would take a rather efficient generator and gearbox combination to have losses less than 6.4 percent of rated power while losses of 12.5 percent would indicate a rather mediocre design. We would expect then that u_c would almost always lie in the range between 0.4 and $0.5u_R$.

Commercial wind turbines typically have furling speeds between 20 and 25 m/s and rated wind speeds between 10 and 15 m/s. A furling speed of twice the rated speed means that the turbine control system is able to maintain a constant power output over an eight to one range of wind power input. This is quite an engineering challenge. This design difficulty plus the difficulty of building wind turbines which can survive operation in wind speeds greater than perhaps 25 m/s means that the furling speed will not normally be above $2u_R$, unless u_R happens to be chosen unusually low for a special application.

We can see from this discussion that selecting a rated wind speed u_R is an important part of wind turbine design. This selection basically determines the cut-in speed and also imposes certain constraints on the furling speed. As stated earlier, we want to select u_R so

the average power will be as large as possible for a given turbine area. The capital investment in the turbine will be proportional to the turbine area so maximizing the average power will minimize the cost per unit of energy produced. If the rated speed is chosen too low, we will lose too much of the energy in the higher speed winds. If the rated speed is too high, the turbine will seldom operate at capacity and will lose too much of the energy in the lower speed winds. This means that the average power output will reach a maximum at a specific value of rated wind speed. We can determine this value by evaluating Eq. 30 for various values of u_R and P_{eR} .

We can gain some insight into this design step by normalizing Eq. 30. We first observe that the quantity inside the brackets of Eq. 30 is called the *capacity factor* CF. Also called the *plant factor*, it is an important design item in addition to the average power.

When we combine Eqs. 15, 16, and 30 we get

$$P_{e,ave} = P_{eR}(\text{CF}) = \eta_o \frac{\rho}{2} A u_R^3 (\text{CF}) \quad \text{W} \quad (31)$$

The choice of rated wind speed will not depend on the rated overall efficiency, the air density, or the turbine area, so these quantities can be normalized out. Also, since the capacity factor is expressed entirely in normalized wind speeds, it is convenient to do likewise in normalizing Eq. 31 by dividing the expression by c^3 to get the term $(u_R/c)^3$. We therefore define a normalized average power P_N as

$$P_N = \frac{P_{e,ave}}{\eta_o(\rho/2)Ac^3} = (\text{CF}) \left(\frac{u_R}{c} \right)^3 \quad (32)$$

Plots of P_N are given in Fig. 17 for various values of the Weibull shape parameter k and for two ratios of cut-in to rated speed. As argued earlier, most turbines will have cut-in speeds between 0.4 and 0.5 of the rated wind speed, so these plots should bracket the designs of practical interest.

We see that maximum power is reached at different values of u_R/c for different values of k . For $u_c = 0.5u_R$, the maximum power point varies from $u_R/c = 1.5$ to 2.5 as k decreases from 2.6 to 1.4. As the cut-in speed is lowered to $0.4u_R$, the maximum power point varies from $u_R/c = 1.6$ to 3.0 . If $k = 2$ at a particular site, the optimum value of u_R/c is between 1.8 and 2.0. We saw in Chapter 2 that c is usually about 12 percent larger than the mean wind speed, so the optimum design for energy production is a rated speed of about twice the mean speed. If the mean wind speed at a site is 6 m/s, then the rated speed of the turbine should be about 12 m/s.

This design choice only holds for wind regimes where k is about 2. In a trade wind regime, k will be significantly larger than 2, so a rated speed perhaps 1.3 times the mean speed may be a better choice in such locations.

We see that the curves for P_N are gently rounded near their maximum values so small

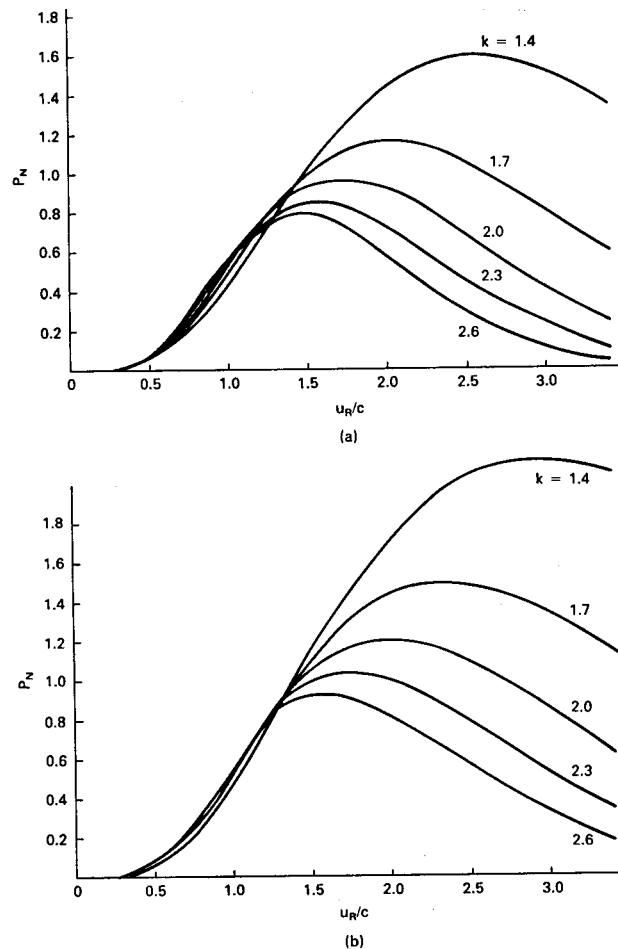


Figure 17: Normalized power versus normalized rated speed: (a) $u_c = 0.5u_R$, $u_F = 2u_R$; (b) $u_c = 0.4u_R$, $u_F = 2u_R$.

errors in selecting a rated speed are not critical. In fact, a manufacturer could cover most of the potential market by having only two rated speeds for a given size of turbine. A rated speed of 11 m/s would be adequate for most sites with mean wind speeds up to 6 m/s, and a rated speed of 13 or 14 m/s would be appropriate for sites with greater wind speeds. This is a big help to the mass production of turbines in that it is not essential to have a turbine specifically designed for each site. Only when a wind turbine factory is dedicated to producing turbines for a specific wind regime, such as a large wind farm, would a more detailed design be advisable.

It would appear from Fig. 17 that sites with lower k are superior to those with larger k . This is true only if the mean wind speed is the same at each site. As was mentioned in Chapter 2, sites with low mean wind speeds tend to have lower values of k than sites with greater mean wind speeds. These lower wind speeds will usually reduce the average power

more than the increase due to lower values of k . However, if two sites have the same mean wind speed, the site with the lower k will have the larger energy production.

Once we select u_R/c to maximize the average power, we can find the rated power for a turbine with a given area and rated overall efficiency located at an elevation with a known average air density. We know that

$$\text{energy} = (\text{average power})(\text{time}) \quad (33)$$

Therefore, the yearly energy production of such a turbine is

$$W = P_{e,\text{ave}}(\text{time}) = (\text{CF})P_{eR}(8760) \quad \text{kWh} \quad (34)$$

where 8760 is the number of hours in a year of 365 days and P_{eR} is expressed in kilowatts.

We note that when we select a larger value of u_R for a turbine that the rated power P_{eR} will increase. This is accompanied by a decrease in capacity factor CF. This decrease has economic implications which may force us to select a smaller rated speed than that which produces maximum energy. What we really want is the maximum energy production per dollar of investment, which may yield a different design than the one which strictly maximizes total energy. As we increase P_{eR} for a given turbine, the costs of the necessary generator, transformer, switches, circuit breakers, and distribution lines all increase. However, the decrease in capacity factor means that these items are being used proportionately less of the time. Equipment costs will increase more rapidly than energy output as we approach the peak of the curves in Fig. 17 so the actual economic optimum will be at a rated wind speed slightly below that which yields maximum yearly energy.

These economic considerations may extend well beyond the equipment immediately attached to the wind turbine. Wind turbines with low capacity factors supply power to the utility grid in an intermittent fashion, which forces conventional generating plants to cycle more than they otherwise would. This cycling of conventional generating plants causes them to operate at lower efficiencies than if operated at more constant power levels, so the economic optimum when the entire power network is considered may be at an even lower rated wind speed and higher capacity factor. A proper determination of the rated wind speed for this overall economic optimum may require a very detailed study of the power system. Lacking such a detailed study, a reasonable design procedure would be to use the u_R/c ratio at which the normalized power is perhaps 90 percent of the peak normalized power for a given wind regime. This will yield a total energy production close to the maximum, at a much better capacity factor.

It is of some interest to actually examine the variation in capacity factor with u_R/c . A plot of capacity factor versus u_R/c for $u_c = 0.5u_R$ and $k = 2$ is given in Fig. 18. The curve of practical interest is for $u_F = 2u_R$, but the curve for $u_F = 5u_R$ is also shown. There is essentially no difference between the curves for $u_R/c \geq 1$ but significant differences appear for very low values of rated wind speed. It is seen that the capacity factor does not exceed 0.6

for the curve of $u_F = 2u_R$. There is no combination of practical values for cut-in, rated, and furling speeds which will yield a capacity factor greater than 0.6 in a wind regime described by the Weibull shape parameter $k = 2$. The average power will never be more than 0.6 of the rated power in such a wind regime. Only if impractical values of rated and cut-in speeds are selected can the capacity factor be raised above 0.6. For example, if we have a good wind regime described by $c = 10$ m/s (mean speed $\bar{u} = 9$ m/s) we could have a capacity factor approaching 0.9 if we pick a rated wind speed of 4 m/s ($u_R/c = 0.4$) and if the turbine could deliver rated power up to $u_F = 50$ m/s. Even if this were technically possible, it would not be economically practical. We shall see that economics will normally force us to a rated wind speed greater than c , in which case a furling speed of approximately twice the value of c will produce the same capacity factor as a larger furling speed. This is true because the wind rarely blows at speeds greater than $2c$, so wind speeds above $2c$ do not significantly affect the average power.

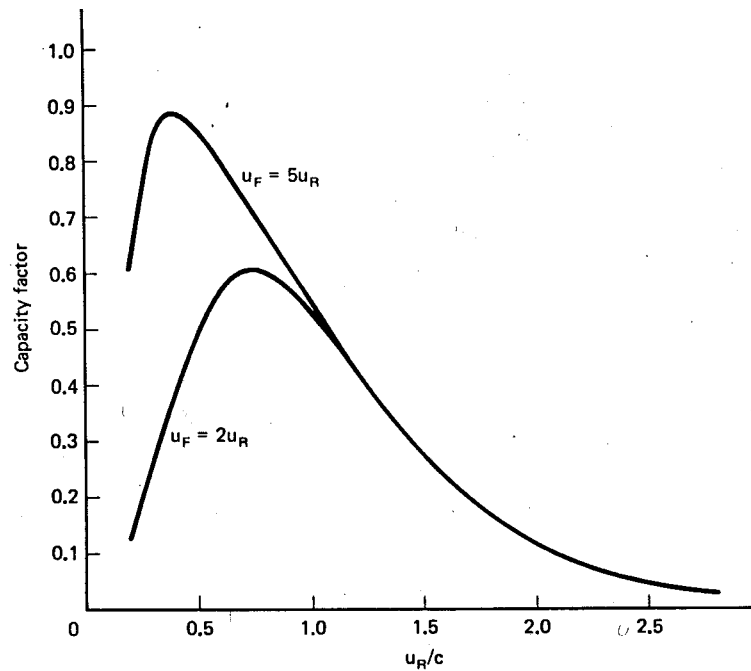


Figure 18: Wind turbine capacity factor as a function of rated speed. $u_c = 0.5u_R$ and $k = 2$.

The important point is that the capacity factor decreases rapidly with increasing values of rated wind speed for practical values of u_R . As the rated wind speed is increased, the turbine will operate fewer hours at rated power and more hours at partial power or below cut-in. This decrease in capacity factor must be balanced against an increase in total energy production to obtain the desired economic optimum.

Example

Preliminary data suggest that the 50 m wind speeds at a potential wind farm site are characterized

by the Weibull parameters $c = 9$ m/s and $k = 2.3$. You work for a wind farm company that plans to build wind machines of the same size as the MOD-2 (rotor diameter 91.5 m) but optimized for this site, if necessary. You know that the MOD-2 has a rated power of 2500 kW at a rated wind speed of 12.4 m/s at hub height. You conservatively estimate that $u_c = 0.5u_R$ and $u_F = 2u_R$.

- a) What is the optimum rated wind speed?
- b) What is the capacity factor of your optimized turbine?
- c) What are the average power and yearly energy production values for your optimized turbine?
- d) What would be the capacity factor, average power, and yearly energy production of the MOD-2 turbine used in that wind regime without modification?
- e) Should you recommend building the MOD-2 on this site without modification?

From Fig. 17 we see that the normalized power is greatest at $u_R/c = 1.6$ for $k = 2.3$ and $u_c = 0.5u_R$. The optimum rated wind speed is then

$$u_R = 1.6(9) = 14.4 \text{ m/s}$$

The capacity factor is, from Eq. 30,

$$\begin{aligned} \text{CF} &= \frac{\exp[-(1.6/2)^{2.3}] - \exp(-1.6)^{2.3}}{(1.6)^{2.3} - (1.6/2)^{2.3}} - \exp\{-[2(1.6)]^{2.3}\} \\ &= \frac{0.550 - 0.052}{2.948 - 0.599} - 5 \times 10^{-7} \\ &= 0.212 \end{aligned}$$

The rated power, assuming all efficiencies remain the same, will just be in the ratio of the cube of the wind speeds.

$$P_{eR} = 2500 \left(\frac{14.4}{12.4} \right)^3 = 3900 \text{ kW}$$

The average power is

$$P_{e,ave} = (\text{CF})P_{eR} = (0.212)(3900) = 830 \text{ kW}$$

The yearly energy production is then

$$W = 830(8760) = 7,270,000 \text{ kWh}$$

The same computations for the unmodified MOD-2 in that wind regime yield the following results:

$$\begin{aligned}
 \text{CF} &= 0.319 \\
 P_{e,\text{ave}} &= 800 \text{ kW} \\
 W &= 7,000,000 \text{ kWh}
 \end{aligned}$$

Optimizing the MOD-2 for this site has increased our total energy production about 4 percent while increasing the rated power by 56 percent. The increase in total energy is desirable, but only if it can be accomplished in a cost effective manner. If the basic MOD-2 structure is adequate to handle the larger power rating without structural changes, then we can get 4 percent more energy for perhaps 1 percent greater investment in the electrical system. If the structure needs to be changed, however, the additional cost could easily exceed the additional benefit.

Another difficulty seen in this example is the difference between the capacity factors. The capacity factor for the standard MOD-2 is 0.319 while that for the optimized system is only 0.212. This lower capacity factor means that the machine will be operating in a more intermittent fashion and this poses additional operating difficulties for the other generating plants on the system, as mentioned earlier. It may well be that the best decision is to use the standard MOD-2 without any effort to optimize it.

It should now be evident that rated power is not a totally satisfactory parameter for distinguishing between wind turbines. We can put a larger generator on a given set of blades and actually reduce the yearly energy production. We also reduce the capacity factor, which may be an important factor in some situations. Several pieces of information are needed to properly specify a wind turbine, including average power and capacity factor in a variety of wind regimes. Specifying only the rated power makes it difficult to properly compare competing turbines.

6 TORQUE AT CONSTANT SPEEDS

Most wind turbines extract power from the wind in mechanical form and transmit it to the load by rotating shafts. These shafts must be properly designed to transmit this power. When power is being transmitted through a shaft, a *torque* T will be present. This torque is given by

$$T = \frac{P}{\omega} \quad \text{N} \cdot \text{m}/\text{rad} \quad (35)$$

where P is mechanical power in watts and ω is angular velocity in rad/sec. The torque in the low speed shaft of Fig. 12 is $T_m = P_m/\omega_m$ while the torque in the high speed shaft is $T_t = P_t/\omega_t$. The units may be expressed as either N·m/rad or N·m, depending on one's preference. We shall express torque in rotating shafts in N·m/rad and torque on a stationary structure such as a tower in N·m. This will hopefully clarify the application and make the necessary analysis more obvious.

The application of torque to a shaft causes internal forces or pressures on the shaft material. Such a pressure is called the *stress* f_s with units Pa or N/m². Since this pressure is trying

to *shear* the shaft, as opposed to compress or stretch, it is referred to as the *shearing stress*. The shearing stress varies with the distance from the shaft axis, having the largest value at the surface of the shaft. It is shown in textbooks on Mechanics of Materials that the shearing stress in a solid shaft is given by

$$f_s = \frac{Tr}{J} \quad \text{N/m}^2 \quad (36)$$

where r is the distance from the axis of the shaft to where the stress is to be determined, and J is the *polar moment of inertia* of the shaft. It is given by

$$J = \frac{\pi r_o^4}{2} \quad \text{m}^4 \quad (37)$$

where r_o is the shaft radius.

It should be mentioned that there are two distinct but closely related quantities which are both called the moment of inertia. One is the *area* moment of inertia, with units m^4 , and the other is the *mass* moment of inertia, with units $\text{kg}\cdot\text{m}^2$. The area moment of inertia is used in studying the mechanics of materials, normally in a static or stationary mode, while the mass moment of inertia is used in determining the dynamics of rotating structures. These topics are usually covered in separate textbooks, so the prefixes *area* or *mass* are usually omitted, with the reader expected to know which one is meant by the context. We shall sometimes omit the prefixes also, but we shall use the symbol J for the polar area moment of inertia and the symbol I for the polar mass moment of inertia. We have no need for the rectangular moment of inertia in this text, so we can also drop the word polar from the terminology.

The mass moment of inertia is found from the area moment of inertia by multiplying by the area density ρ_a in kg/m^2 . The area density is measured across the area perpendicular to the axis of rotation.

Example

A solid steel shaft has a radius of 0.1 m and a length of 0.8 m. Find the area moment of inertia J and the mass moment of inertia I if the volume density of steel is $7800 \text{ kg}/\text{m}^3$.

The area moment of inertia is given by Eq. 37 as

$$J = \frac{\pi(0.1)^4}{2} = 1.57 \times 10^{-4} \text{ m}^4$$

The area density of the shaft would simply be the length times the volume density.

$$\rho_a = 0.8\rho = 0.8(7800) = 6240 \text{ kg}/\text{m}^2$$

The mass moment of inertia is then

$$I = J\rho_a = 1.57 \times 10^{-4}(6240) = 0.980 \text{ kg}\cdot\text{m}^2$$

One way of designing shafts to carry a given torque is to select a maximum shearing stress which will be allowed for a given shaft material. This stress occurs at $r = r_o$, so Eqs. 36 and 37 can be solved for the shaft radius. The shaft diameter which will have this maximum stress is

$$D = 2r_o = 2\sqrt[3]{\frac{2T}{\pi f_s}} \quad \text{m} \quad (38)$$

The maximum stress in Eq. 38 is usually selected with a significant safety factor. Recommended maximum stresses for various shaft materials can be found in machine design books.

Example

You are designing a wind turbine with an electrical generator rated at 200 kW output. The low speed shaft rotates at 40 r/min and the high speed shaft rotates at 1800 r/min. Solid steel shafts are available with recommended maximum stresses of 55 MPa. The gearbox efficiency at rated conditions is 0.94 and the generator efficiency is 0.93. Determine the necessary shaft diameters.

From Eq. 11, the angular velocities for the low and high speed shafts are

$$\omega_m = \frac{2\pi(40)}{60} = 4.19 \text{ rad/s}$$

$$\omega_t = \frac{2\pi(1800)}{60} = 188.5 \text{ rad/s}$$

The power in the high speed shaft is

$$P_t = \frac{200,000}{0.93} = 215,000 \text{ W}$$

The power in the low speed shaft is

$$P_m = \frac{215,000}{0.94} = 229,000 \text{ W}$$

The torques are then

$$T_m = \frac{229,000}{4.19} = 54,650 \text{ N} \cdot \text{m/rad}$$

$$T_t = \frac{215,000}{188.5} = 1140 \text{ N} \cdot \text{m/rad}$$

The shaft diameters are then computed from Eq. 37.

$$D_L = 2 \sqrt[3]{\frac{2(54,650)}{\pi(55 \times 10^6)}} = 0.172 \text{ m}$$

$$D_H = 2 \sqrt[3]{\frac{2(1140)}{\pi(55 \times 10^6)}} = 0.0473 \text{ m}$$

It can be seen that the low speed shaft is rather substantial in size. This adds to the mass and cost of the turbine and should be held to a minimum length for this reason.

Torque at the rotor shaft will vary significantly as the rotor goes by the tower. This will be smoothed out somewhat by the inertia and damping of the system but will still appear in the electrical power output curve. Fig. 19 illustrates this situation for the MOD-0 wind turbine in a 15 m/s wind[7]. The system losses have been subtracted from the power input curve, so the areas under the input and output curves are the same. The actual aerodynamic rotor input power is rather difficult to measure, so its curve is theoretically developed. It shows the input power decreasing to 40 kW as a blade goes by the tower and increasing to 120 kW as the blade clears the tower. The torque will follow the same pattern since the rotor rotational speed is fixed. The output power is considerably damped, but still shows a variation of about 18 kW for a stiff steel shaft, 16 kW for a flexible elastomeric shaft and 14 kW for a fluid coupling. The system lag is such that the output power is at a peak when the rotor power is at a minimum.

A power variation of this magnitude can be a major problem to a utility. It can affect voltage levels, causing lights to flicker. It can cause utility control equipment such as voltage regulators to cycle excessively. Careful attention must be given to the design of the drive train in order to hold this variation to a minimum.

This power flow variation can also be minimized by placing several wind turbines in a wind farm in parallel operation. The larger wind turbines normally use synchronous generators, to be discussed in the next chapter. One feature of synchronous generators in parallel is that they all turn at exactly the same speed, and the angular positions of their shafts vary only slightly with individual power flows. If fixed gearing is used, and there are no drive train components like vee-belts or fluid couplings which allow slip, each rotor in the wind farm can be at a different angular position. A collection of 18 turbines with a 10° angular position difference between individual rotors would be expected to have a much smoother net output than the output of any one turbine.

The torque and power variation for a Darrieus turbine is even more pronounced than that for a horizontal axis turbine. Figure 20 shows the aerodynamic torque for the Sandia 17-m

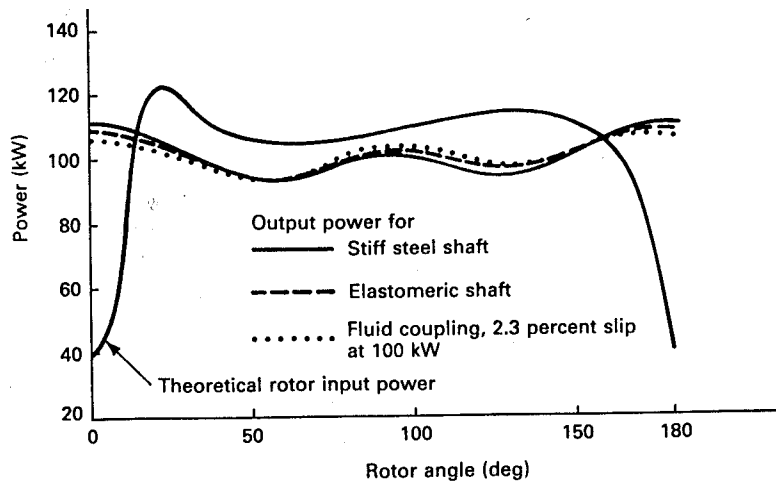


Figure 19: MOD-0 power output for three high-speed shaft configurations.

Darrieus at a rotational speed of 50.6 r/min and at wind speeds of 9.8, 15.2, and 19.7 m/s. These are measurements of the actual torque caused by the wind, obtained by a clever use of accelerometers on the blades[6]. The shaft torque measured by torque sensors is much smoother. As expected, the two bladed machine has two distinct torque cycles per rotor revolution. At a wind speed of 9.8 m/s, the aerodynamic torque peaks at a rotor angle just below 90° , as defined in Fig. 21, at which point the plane of the rotor is parallel to the wind. The torque variation at this wind speed is nearly symmetric with changes in angular position and goes slightly negative when the plane of the rotor is perpendicular to the direction of the wind.

As the wind speed increases the torque pattern becomes more complex. We saw in Fig. 15 that the power output of this Darrieus does not increase above a certain point, even though the power in the wind continues to increase with wind speed. We now see in Fig. 20 that the average torque at two wind speeds may be about the same, but that the instantaneous torque of the higher wind speed may oscillate more widely. This is due to complex interactions between the blades, the supporting tower, and the air flow, which we shall not try to explain. The important point to note is that there is a cyclic torque variation in both the horizontal and vertical axis turbines and that the drive train needs to be designed with this torque variation in mind.

7 DRIVE TRAIN OSCILLATIONS

When torque is applied to a shaft, it will twist. This is illustrated in Fig. 22 where the line AB on a shaft of length L has been twisted to position AC . The total twist is the angle θ . The twist will be directly proportional to the torque as long as the material remains in its

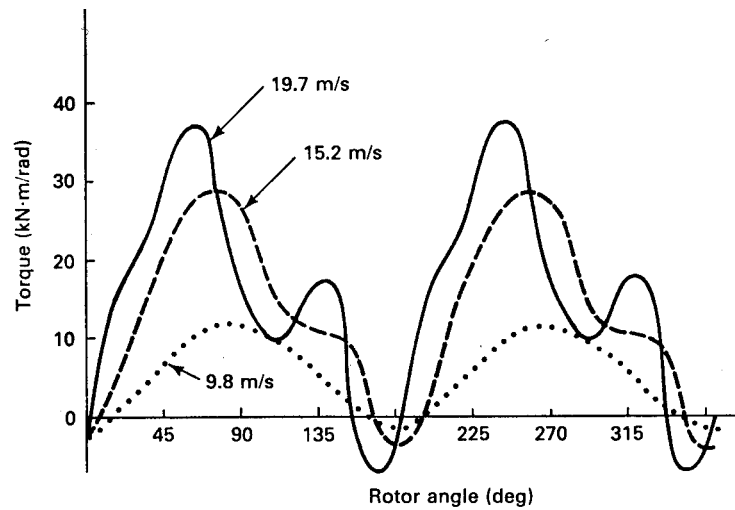


Figure 20: Aerodynamic torque variation for Sandia 17-m Darrieus at 50.6 r/min.

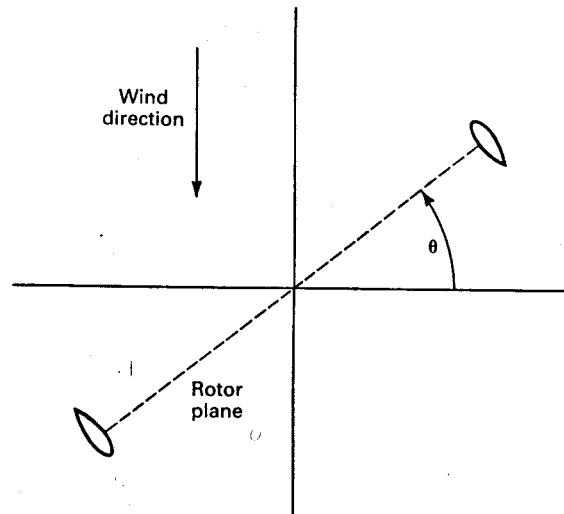


Figure 21: Definition of rotor angle for Sandia 17-m Darrieus.

elastic range. Permanent deformation occurs when a material exceeds its elastic range.

The shaft can be thought of as a spring with a *torsional spring constant* k_T where

$$k_T = \frac{T}{\theta} \quad (39)$$

The angle θ has to be expressed in radians, of course. A large value of k_T represents a

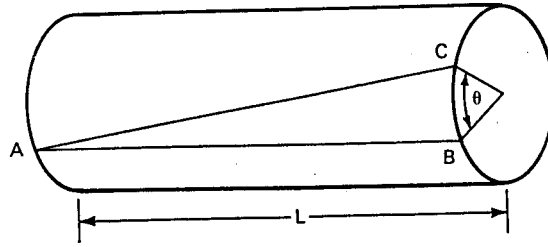


Figure 22: Shaft twisted under an applied torque.

stiff shaft, while a small value represents a soft or flexible shaft.

The torsional spring constant is also given by

$$k_T = \frac{JG}{L} \quad (40)$$

where J is the polar area moment of inertia, G is the *shear modulus*, and L is the length of the shaft. The shear modulus is the proportionality constant between a shear stress and the resulting deflection or *strain*. A typical value for the shear modulus for steel is 83 GPa (83×10^9 Pa).

A twisted shaft contains potential energy, just like a compressed spring. The amount of this potential energy is given by

$$U = \frac{k_T \theta^2}{2} \quad \text{J} \quad (41)$$

This potential energy has to be supplied to the shaft during system start-up and will be delivered back to the system during shut-down. Also, when a wind gust strikes the turbine, part of the extra power will go into shaft potential energy rather than instantly appearing in the electrical output. This stored energy will then go from the shaft into the electrical system during a wind lull. We see then that a shaft helps to smooth out the power fluctuations in the wind.

Example

Assume that the high speed shaft of a wind turbine has a torque of 1140 N·m/rad, an angular velocity of 188.5 rad/s, a diameter of 0.0473 m, and a shear modulus of 83 GPa. The length is 2 m. Find the rotation angle θ and the energy stored in the shaft.

From Eq. 37 the moment of inertia is

$$J = \frac{\pi(0.0473/2)^4}{2} = 4.914 \times 10^{-7} \text{ m}^4$$

From Eq. 40, the torsional spring constant is

$$k_T = \frac{(4.914 \times 10^{-7})(83 \times 10^9)}{2} = 20,400$$

From Eq. 39, the angle is

$$\theta = \frac{T}{k_T} = \frac{1140}{20,400} = 0.0559 \text{ rad} = 3.20^\circ$$

The potential energy is then given by

$$U = \frac{(20,400)(0.0559)^2}{2} = 31.9 \text{ J}$$

The amounts of twist and stored potential energy in this example are not large and will cause no problems in steady state operation. Operation is never quite steady state, however, because of variations in wind speed and direction, and the tower shadow experienced by each blade once per revolution. The shaft acts as a spring connecting two rotating masses (the blades on one end and the generator on the other end) and this system can oscillate in a torsional mode. If the oscillation frequency happens to be the same as that of the pulse from the tower shadow, the system will oscillate with ever increasing amplitude until the shaft breaks or some protective circuit shuts the turbine down. This means that a shaft which is conservatively designed for steady state operation may fail catastrophically as soon as it is placed in operation. We therefore need to know the frequency of oscillation to make sure this does not happen.

A simple model for a torsional oscillator[5] is shown in Fig. 23. One end of the shaft is attached to a rigid support and the other end is attached to a disk with a mass moment of inertia I . If the disk is displaced through an angle θ , a restoring torque T is exerted on the disk by the shaft, of magnitude $T = k_T\theta$. If the disk is released, the restoring torque T results in angular acceleration of the disk, which causes rotation of the disk back toward the equilibrium position. In this process, the potential energy stored in the shaft is transformed to rotational kinetic energy of the disk. As the disk reaches its equilibrium position, the kinetic energy acquired causes the disk to overshoot the equilibrium position, and the process of energy transformation reverses, creating oscillations of the disk.

The basic equation relating torque to angular acceleration α for this simple torsional oscillator is

$$-k_T\theta = I\alpha \quad (42)$$

The minus sign is necessary because the restoring torque is opposite to the angular displacement. We can now replace α by $d^2\theta/dt^2$ to get the second order differential equation

$$\frac{d^2\theta}{dt^2} + \left(\frac{k_T}{I}\right)\theta = 0 \quad (43)$$

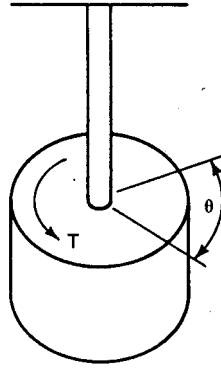


Figure 23: Simple torsional oscillator.

The solution to this equation is

$$\theta(t) = A \sin \omega t + B \cos \omega t \quad (44)$$

where A and B are constants to be determined from the initial conditions. The radian frequency of oscillation is given by

$$\omega = \sqrt{\frac{k_T}{I}} \quad \text{rad/s} \quad (45)$$

We see that the frequency of oscillation is directly proportional to the torsional spring constant and inversely proportional to the inertia of the disk. If the shaft is rotating rather than fixed, then ω of Eq. 45 is the frequency of oscillation about the mean shaft speed.

Example

The turbine in the previous example suddenly loses the interconnection to the electrical grid, thus allowing the high speed shaft to unwind. The inertia of the generator is $10.8 \text{ kg}\cdot\text{m}^2$, which is so much smaller than the inertia of the turbine blades and gearbox that the generator acts like a torsional oscillator with the other end of the high speed shaft fixed. Find an expression for θ as a function of time.

From the previous example we can take $\theta(0+) = 0.0559 \text{ rad}$. The relative angular velocity $d\theta/dt$ can not change instantaneously, so

$$\frac{d\theta(0+)}{dt} = \frac{d\theta(0-)}{dt} = 0.$$

We can then insert these two initial conditions into Eq. 44 and evaluate the constants.

$$\theta(0+) = 0.0559 = A \sin 0 + B \cos 0$$

$$\frac{d\theta(0+)}{dt} = 0 = A\omega \cos 0 + B\omega(-\sin 0)$$

From these two equations we observe that $B = 0.0559$ and $A = 0$.

The relative angular velocity about the mean angular velocity is given by Eq. 45.

$$\omega = \sqrt{\frac{20,400}{10.8}} = 43.46 \text{ rad/s}$$

The expression for θ is then

$$\theta = 0.0559 \cos 43.46t \text{ rad}$$

The generator will oscillate with respect to the gearbox at the rate of 43.46 rad/s or 6.92 Hz. The amplitude is not large but the torque reversal twice per cycle would probably produce audible noise.

An actual wind turbine drive train is quite complicated[7]. The rotor itself is not a perfect rigid body, but is able to flex back and forth in the plane of rotation. The rotor can therefore be modeled as a stiff shaft supplying power to the rotor hub. We have the various inertias of the rotor blades, hub, gearbox, generator, and shafts. There is *damping* caused by the wind, the oil in the gearbox, and various nonlinear elements. This damping causes any oscillations to die out if they are not being continually reinforced. A reasonably complete model[7] may have six inertial masses separated by five shafts and described by five second order differential equations. These equations are all coupled so the solution process requires a computer program. The full solution contains a number of oscillation frequencies, some of which may be heavily damped and others rather lightly damped. If the system is pulsed at the lightly damped oscillation frequencies, serious damage can occur.

The most important source of a pulsation in the driving function is that of the rotor blades passing by the tower each revolution. If we had only one blade on the rotor, we would have one pulse per revolution. If the rotor were spinning at 40 r/min, the pulsation frequency seen by the shaft would be 40 pulses per minute, or 40/60 pulses per second, 0.667 Hz. A rotor with two perfectly identical blades will have the lowest pulsation frequency equal to two pulses per revolution, or 1.33 Hz for a 40 r/min rotor. In practice, the two blades are not identical, so both 0.667 and 1.33 Hz would be available to drive oscillations near those frequencies. These driving frequencies are normally referred to as 1P and 2P. Oscillation frequencies near a multiple of the driving frequencies can also be excited, especially 4P, 6P, 8P, and so on.

Example

The original low speed steel shaft for the MOD-0 wind turbine had a torsional spring constant $k_T = 2.4 \times 10^6$ N·m/rad. The inertia of the rotor and hub is 130,000 kg·m². What is the frequency of oscillation, assuming the generator connected to the electrical grid causes a high effective inertia at the gearbox end of the low speed shaft, so the torsional oscillator model applies?

From Eq. 45 we find

$$\omega = \sqrt{\frac{2.4 \times 10^6}{130,000}} = 4.297 \text{ rad/s}$$

The frequency is then

$$f = \frac{\omega}{2\pi} = 0.684 \text{ Hz}$$

Reducing the actual system to a single inertia and a single torsional spring is a rather extreme approximation, so any results need to be viewed with caution. The actual system will have many modes of oscillation which can be determined by a computer analysis, only one of which can be found by this approximation.

The oscillation frequency found in the above example would appear to be rather close to the pulsation frequency of 0.667 Hz. In fact, when the MOD-0 was first put into service, oscillations at this frequency were rather severe. It was discovered that the two blades were pitched differently by 1.7 degrees. Even with this asymmetry corrected, turbulent winds would still cause oscillations. It was therefore decided to consider two other shaft combinations. One of these was an elastomeric shaft on the high speed side of the gearbox with a torsion spring constant of about 3000 N·m/rad. The other was a fluid coupling set to slip 2.3 percent when the transmitted power is 100 kW. The fluid coupling dissipates 2.3 percent of the power delivered to it, but adds sufficient damping to prevent most drive train oscillations that would otherwise be present. It was determined that both modifications would reduce the oscillations to acceptable levels.

This type of problem is typical with new pieces of equipment. We have excellent hindsight but our foresight is not as good. The only way we can be positive we have correctly considered all the vibration modes is to build a turbine and test it. This was one of the advantages of the MOD-0, in that it served as a test bed which permitted a number of such problems to be discovered and corrected.

8 STARTING A DARRIEUS TURBINE

A Darrieus wind turbine is not normally self starting, so some mechanism for starting must be used. This mechanism may be direct mechanical, hydraulic, or electrical, with electrical being preferred when utility power is available. As will be discussed in the next chapter, the induction machine will work as a motor for starting purposes, and then automatically change role and become a generator as the Darrieus accelerates. This is a convenient and economical method of starting the turbine.

Starting a high inertia load such as a large Darrieus turbine requires careful design to assure that adequate but not excessive torque is available for a sufficient time to start the turbine without damaging the electrical equipment. This design requires that we know the acceptable turbine acceleration and the required energy to get the turbine to its operating speed.

A rotating mass with a moment of inertia I has a stored energy

$$U = \frac{I\omega^2}{2} \quad \text{J} \quad (46)$$

where ω is the angular velocity in rad/s and I has units kg·m². The motor must supply this amount of energy to the rotor without exceeding the rated torque of the drive train.

The acceleration of the rotor when a torque T is applied is

$$\alpha = \frac{T}{I} \quad \text{rad/s}^2 \quad (47)$$

The time t_s required for starting the turbine and accelerating it to rated angular velocity ω_R with rated torque T_R would be

$$t_s = \frac{\omega_R}{\alpha} = \frac{\omega_R I}{T_R} \quad \text{s} \quad (48)$$

Example

A point design[4] for a 120-kW Darrieus has a moment of inertia $I = 51,800$ kg·m² and rated torque of 23,900 N·m/rad. Rated rotor speed is 52 r/min. How much time is required to accelerate the rotor to rated speed, assuming rated torque is applied and that there is no help from the wind?

From Eq. 48, the time required is

$$t_s = \frac{\omega_R I}{T_R} = \frac{2\pi(52/60)(51,800)}{23,900} = 11.8 \text{ s}$$

If the turbine geometry, tip speed ratio, and mass of the blades per unit length are all fixed, then the inertia and the rated torque increase in direct proportion to each other as the turbine size is increased. When these conditions hold, the starting time is the same for a larger turbine as it is for a smaller one. Actually, the blade mass per unit length increases with turbine size, so larger turbines take somewhat longer to start than smaller ones. A 1600-kW Darrieus point design[4] would take 18 seconds to start, as compared with 11.8 s for a 120-kW system that we saw in the previous example.

These starting times are quite long when compared with normal motor starting times, and it is not desirable to start such heavy loads with a directly coupled induction motor. The preferred design is to start the motor while unloaded and then start the Darrieus with a

controlled torque clutch, as shown in Fig. 24. The motor can deliver rated torque at its rated speed and current during any required starting time with no hazard to the motor.

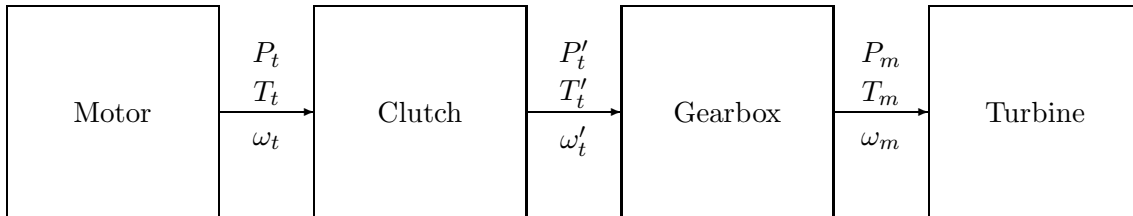


Figure 24: Electric motor and clutch for starting a Darrieus turbine.

The clutch normally is operated with the output torque T'_t equal to the input torque T_t . When the clutch is first engaged, the output angular velocity ω'_t will be zero while the input angular velocity is the rated angular velocity of the motor.

A constant torque applied to the turbine will result in a constant acceleration, as seen from Eq. 48. Since the acceleration is $\alpha = d\omega/dt$, we can determine that the angular velocity of the turbine must increase linearly with time until the turbine reaches its rated angular velocity ω_{mR} . This is shown in Fig. 25.

The power input to the turbine, $P_m = T_m\omega_m$, will also increase linearly with time. The clutch power output P'_t will increase in the same fashion if the gearbox is lossless or if the losses increase linearly with speed. For the purposes of determining clutch sizes, an assumption of a lossless gearbox is usually acceptable. The clutch would normally be placed on the high speed side of the gearbox to minimize the torque which it must transmit.

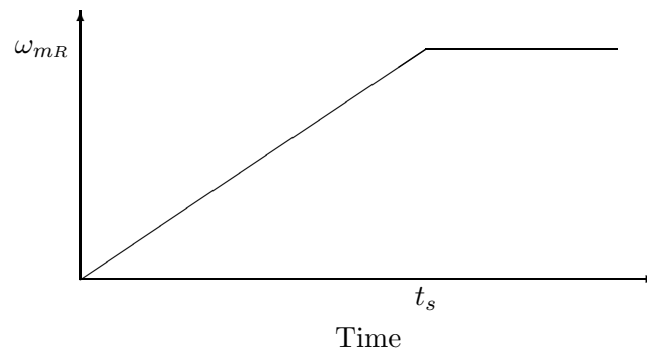


Figure 25: Angular velocity of a turbine with constant torque applied during start.

The energy supplied to the turbine during start is the integral of power, $T\omega$, through time.

$$U = \int_0^{t_s} T_m \omega_m dt = \int_0^{t_s} T_m \omega_{mR} \frac{t}{t_s} dt = \frac{T_m \omega_m t_s}{2} \quad (49)$$

The power input to the clutch is a constant, $P_t = P_{mR} = T_m \omega_{mR}$, during start, so the total energy delivered to the clutch during start is $T_m \omega_{mR} t_s$, or twice the energy delivered to the turbine. This means that the clutch must absorb the same amount of energy during start as the final rotational energy of the turbine. It must do this without mechanical damage due to overheating, hence must be properly sized. Overheating may result in a lowered clutch friction as well as mechanical damage, so the allowable temperature rise in a clutch may be restricted to a rather moderate amount, like 100°C , to assure proper operation.

The temperature rise in the clutch is inversely proportional to the *specific heat capacity* c_p of the clutch plate material. This is the amount of energy required to raise the temperature of 1 kg of material 1°C . A typical value for the c_p of steel is $527 \text{ J/kg}\cdot^\circ\text{C}$. The required mass of a given clutch is then given by

$$m = \frac{U}{c_p \Delta T} \quad \text{kg} \quad (50)$$

where U is the energy the clutch must dissipate, c_p is the specific heat capacity of the clutch material, and ΔT is the allowable temperature increase.

Example

A steel clutch plate is to be used for starting the Darrieus in the previous example. The specific heat capacity is $527 \text{ J/kg}\cdot^\circ\text{C}$ and the allowable temperature rise is 100°C . The density of steel is $\rho = 7800 \text{ kg/m}^3$. The clutch plate is to be circular with a thickness $L = 1 \text{ inch}$ (2.54 cm). What is the minimum radius of the plate?

The energy to be absorbed by the clutch is

$$U = \frac{I\omega^2}{2} = \frac{51,800[(2\pi)(52/60)]^2}{2} = 768,000 \text{ J}$$

The mass is

$$m = \frac{U}{c_p \Delta T} = \frac{768,000}{527(100)} = 14.57 \text{ kg}$$

The volume V is

$$V = \frac{m}{\rho} = \frac{14.57}{7800} = 1.868 \times 10^{-3} \text{ m}^3$$

The radius r_o is then given by

$$r_o = \sqrt{\frac{V}{\pi L}} = \sqrt{\frac{1.868 \times 10^{-3}}{\pi(0.0254)}} = 0.153 \text{ m}$$

This is obviously not excessively large. In fact, since the cost of such a clutch would be a very small fraction of the total turbine cost, it would normally be built larger than this minimum size to allow a greater safety factor and to permit more frequent starts. A larger clutch plate will not get as hot during start and will radiate this heat to the surroundings more rapidly because of a larger surface area.

A clutch guarantees a smooth start on power lines of any capacity with minimum voltage fluctuations and power flow transients. This allows considerable flexibility in the location of the turbine as far as power line availability is concerned.

Each size of turbine needs careful study to determine the most economical and reliable starting technique. In one study of several point designs¹⁰, Darrieus turbines of 10- and 30-kW ratings were found to have sufficiently low inertias that it would be quite practical to start these turbines without a clutch and with full voltage applied to the induction machine. Sizes of 120 kW or more were found to require a clutch, and sizes of 200 kW or more were found to require reduced voltage starting (discussed in the next chapter) for the induction machine even with the clutch disengaged. Once the motor is running at rated speed and rated voltage, the clutch is engaged and the turbine is started.

We see that starting a Darrieus turbine requires careful design of the starting system. Smaller turbines can be started easily, but larger machines require a clutch and perhaps reduced voltage starting for the motor.

9 TURBINE SHAFT POWER AND TORQUE AT VARIABLE SPEEDS

Most wind turbines operate at fixed rotational speeds except when starting and stopping. This simplifies system operation when using synchronous generators paralleled with the utility grid. It also helps to prevent the turbine from being operated at a speed which will excite a mechanical resonance that might destroy the turbine. However, fixed speed operation means that the maximum coefficient of performance C_{pm} is available only at one particular wind speed. A lower coefficient of performance is observed for all other wind speeds, which reduces the energy output below that which might be expected from variable speed operation. That is, if the turbine speed could be adjusted in relation to the wind speed, a higher average coefficient of performance and a higher average power output could be realized. Variable pitch operation at a fixed speed also helps improve the average coefficient of performance, but this is not feasible for turbines such as the Darrieus. Variable pitch operation also increases complexity and cost, hence may not be the most economical solution for all applications. It is therefore interesting to explore variable speed turbine operation. We shall now examine the variation of shaft power and torque with turbine angular velocity, leaving the discussion of specific methods of generating synchronous power from variable speed turbines to Chapter 6.

We shall proceed by examining the variation of P_m as a function of ω_m , with the wind speed

u as a parameter. We shall use the Sandia 17-m Darrieus turbine for discussion purposes. The shaft power P_m for this turbine as a function of shaft rotational speed n is shown in Fig. 26. P_m is seen to rise to a maximum for each wind speed for a particular value of rotational speed. Higher wind speeds have more power in the wind, and the change in tip speed ratio with increasing wind speed causes the maximum to shift to a higher rotational speed. Maximum power is reached at 38 r/min in a 6 m/s wind and at 76 r/min in a 12 m/s wind. The maximum possible shaft power in a 12 m/s wind is eight times that in a 6 m/s wind, as would be expected from the cubic variation of power with wind speed.

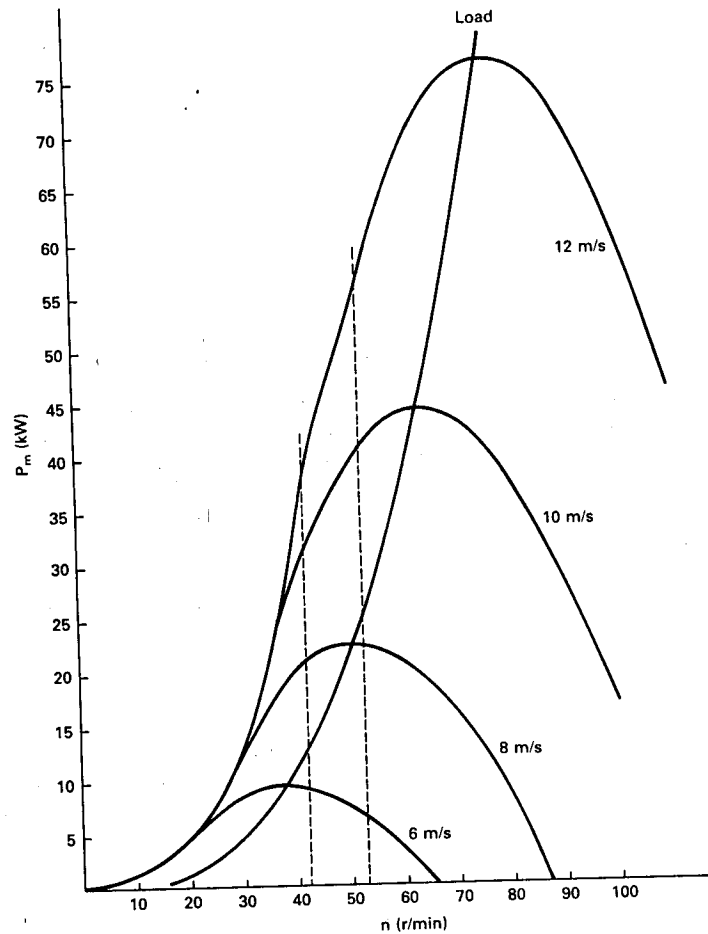


Figure 26: Shaft power output of Sandia 17-m Darrieus in variable-speed operation.

Also shown in Fig. 26 are two vertical dashed lines at 42 and 52.5 r/min. We see that at 42 r/min we are close to the maximum shaft power for $u = 6$ m/s but are below the maximum for higher wind speeds. In fact, no additional shaft power can be obtained for winds above about 12 m/s because curves for higher wind speeds all follow the same line at this rotational speed. At 52.5 r/min we are close to the maximum shaft power at $u = 8$ m/s, and are able to

extract significantly more power from the wind for wind speeds above 8 m/s than was possible at 42 r/min. It would appear that if fixed speed operation is required, that 52.5 r/min is a good choice for sites with a high percentage of winds between 6 and 10 m/s since the turbine can deliver nearly the maximum possible shaft power over this wind speed range. A higher fixed shaft speed would be justified only if there were a significant fraction of wind speeds above 10 m/s. Increasing the shaft speed above 52.5 r/min would decrease the power output for wind speeds below 8 m/s, and this would be a significant penalty at many sites.

Suppose now that the load can accept shaft power according to the curve marked *Load* in Fig. 26 and that the turbine is free to operate at any speed. The turbine will then operate at maximum power for any wind speed, so the energy output will be maximized. If this maximum energy output can be obtained without increasing losses or costs, then we have developed a system which extracts more energy from the wind at a lower cost per unit of energy.

Variable speed operation requires a load which has a suitable curve of power input versus rotational speed. The optimum load will have a cubic variation of input power versus rotational speed. In examining various types of loads, we notice that the input power to pumps and fans often has a cubic variation with rotational speed. The power input to an electrical generator connected to a fixed resistance will vary as the square of the rotational speed. The power input to a generator used to charge batteries will vary even more rapidly than cubic, as we shall see in Chapter 6. Load curves like the one shown in Fig. 26 are therefore quite possible, so variable speed operation is possible with several applications.

In addition to the variation of shaft power with rotational speed, we must also examine the variation of torque with rotational speed. If we take the shaft power curves in Fig. 26 and divide by the angular velocity at each point, we obtain the torque curves shown in Fig. 27. One important feature of these curves is that the torque is zero for $n = 0$, which means that the Darrieus turbine can not be expected to start without help. The friction in the transmission and generator produces an opposing torque, and the torque T_m must exceed this opposing or *tare* torque before the turbine can be accelerated by the power in the wind. The turbine must be accelerated to some minimum rotational speed by an external power source so that T_m will reach a sufficient value to accelerate the turbine up to operating speed. This particular machine would need to be accelerated up to perhaps 10 r/min before the wind would be able to accelerate it to higher speeds.

This external power source is required for reliable starting of any Darrieus turbine. However, there have been cases where a combination of very low tare torques and strong, gusty winds allowed a Darrieus turbine to start on its own, sometimes with disastrous consequences. A Darrieus turbine should have a brake engaged when repairs are being made or at other times when rotation is not desirable.

The torque rises to a maximum at a particular rotational speed for each wind speed, in the same manner as the shaft power. The peak torque is reached at a lower rotational speed than the peak shaft power, as can be seen by a careful comparison of Figs. 26 and 27. From Eq. 35 and the fact that maximum shaft power varies as the cube of the rotational speed,

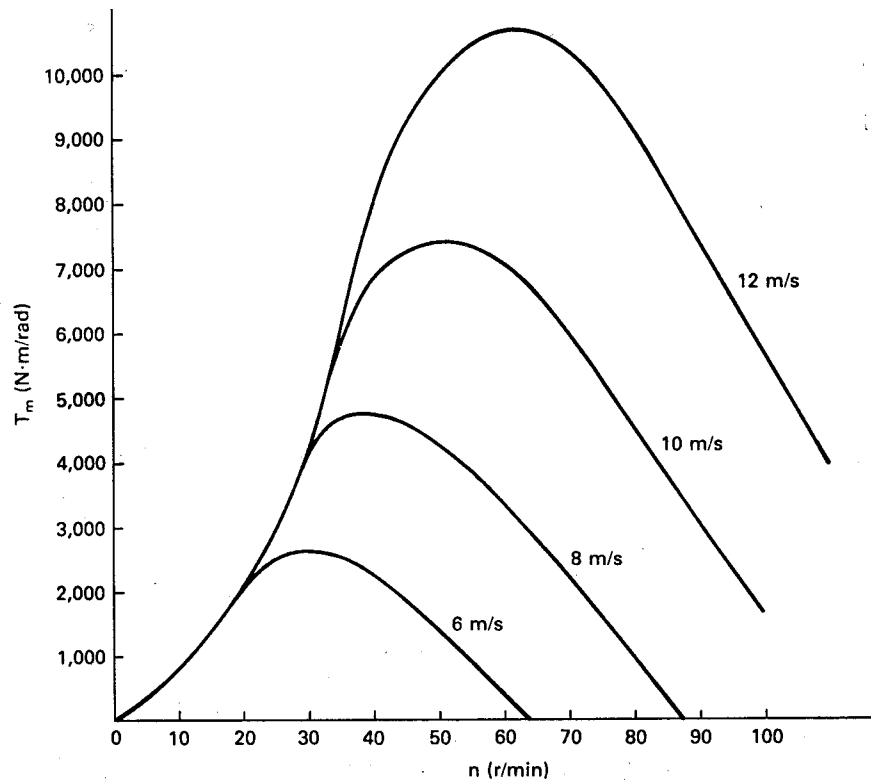


Figure 27: Shaft torque output of Sandia 17-m Darrieus in variable-speed operation.

we can argue that the maximum shaft torque varies as the square of the rotational speed. In Fig. 27, for example, the peak torque in a 12 m/s wind is 10,600 N·m/rad at 60 r/min. The peak torque in a 6 m/s wind is 2650 N·m/rad at 30 r/min. The peak torque has changed by a factor of four while the rotational speed has changed by a factor of two.

The turbine torque T_m must be opposed by an equal and opposite load torque T_L for the turbine to operate at a steady rotational speed. If T_m is greater than T_L , the turbine will accelerate, while if T_m is less than T_L the turbine will decelerate. The mathematical relationship describing this is

$$T_m = T_L + I \frac{d\omega_m}{dt} \quad \text{N} \cdot \text{m/rad} \quad (51)$$

where I is the moment of inertia of the turbine, transmission, and generator, all referred to the turbine shaft.

The relationship between shaft torque and an optimum load torque for the Sandia 17-m Darrieus turbine is illustrated in Fig. 28. We have assumed a load torque with the optimum variation

$$T_L = Kn^2 \quad \text{N} \cdot \text{m}/\text{rad} \quad (52)$$

The constant K is selected so the load torque curve passes through the peaks of the curves for turbine torque at each wind speed.

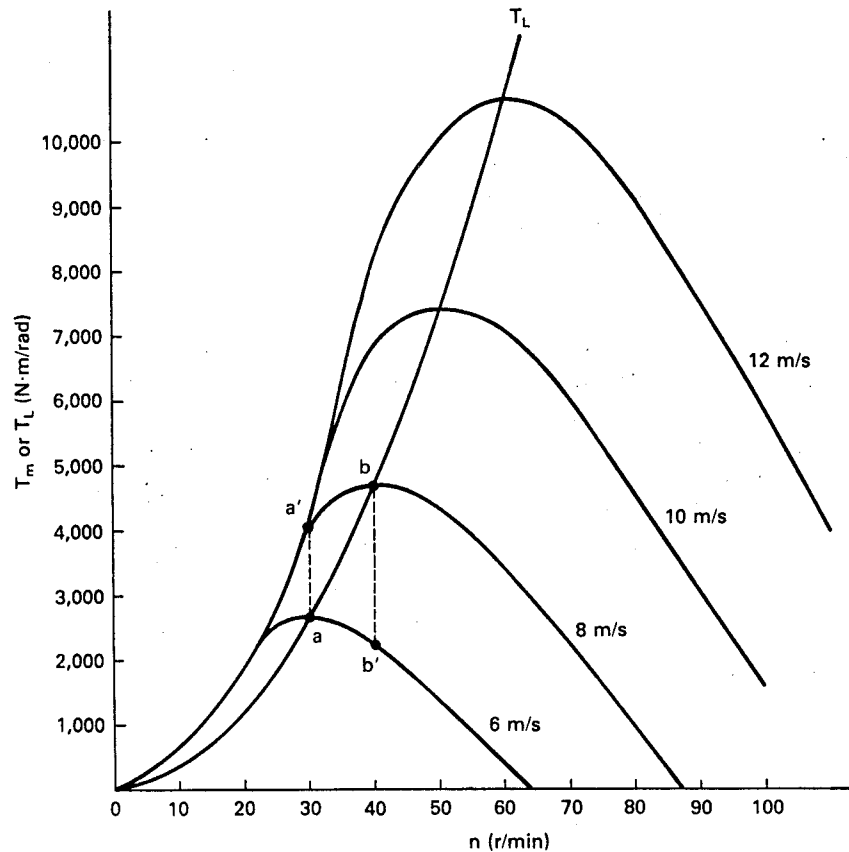


Figure 28: Load with square-law torque variation connected to Sandia 17-m turbine.

In order for the turbine to operate at steady state or at a constant speed, the $d\omega_m/dt$ term of Eq. 51 must be zero, and the load torque must be equal to the shaft torque. Suppose that we have a steady wind of 6 m/s and that the shaft torque and load torque have reached equilibrium at point a in Fig. 28. Now suppose that the wind speed suddenly increases to 8 m/s and remains constant at that speed. The shaft torque T_m increases to the value at a' before the rotational speed has time to change. The load is still requiring the torque at point a . Since the shaft torque is larger than the load torque, the turbine rotational speed will increase until point b is reached, at which time the two torques are equal and steady state has again been reached.

If the turbine is operating at point b and the wind speed suddenly decreases to 6 m/s,

the turbine torque will drop to the value at b' . Since the turbine torque is less than the load torque at this point, the turbine will slow down until the turbine and load torques are again equal at point a .

The constant K of Eq. 52 must be carefully selected for the variable speed system to operate properly. Figure 29 shows the optimum load torque curve for the Sandia 17-m turbine and also shows torque curves for two other loads where the load torque at a given speed has been either doubled or halved. A problem is immediately evident for the load curve described by $2Kn^2$ in that the load torque exceeds the turbine torque above a rotational speed of about 16 r/min. The turbine will not accelerate past this point and the system will be characterized by a rather steady but very low output. A load, such as a pump, with this torque curve would have to be replaced with a load with a smaller rating. Load torque specifications, as well as turbine specifications, are often not precisely known and would be expected to vary slightly between two apparently identical systems, so an adequate margin of safety needs to be applied when selecting load sizes. It is better to slightly undersize a load than to get it so large that the turbine cannot accelerate to rated speed.

The load specified as $Kn^2/2$ requires torques that are just slightly down the turbine torque curve from the maximum torques. The total energy production may be only 10 or 15 percent less for this load than that for the optimum load. This reduction from the optimum output may be acceptable in many applications, except for one major difficulty. The turbine rotational speed in a 12 m/s wind is 80 r/min for the smaller load and 60 r/min for the optimum load. This greater speed may exceed the turbine speed rating and cause some speed limiting system to be activated. We therefore need to have a load with a torque curve that falls between rather narrow limits, perhaps within ten or twenty percent of the optimum torque curve. We shall examine torque curves of specific loads in greater detail later in the text.

Turbines that are self-starting must have greater wind produced torques at low rotational speeds than the Darrieus turbine. The torque at low rotational speeds will be proportional to the *solidity* of the turbine, which is basically the fraction of the projected swept area that is actually covered by the rotor blades. The Savonius turbine would have a solidity of 1.0, for example, with the American Multiblade turbine approaching that value. Two-bladed horizontal axis turbines may have solidities closer to 0.1 at the other extreme. The variation of torque with rotational speed for a low solidity, 10 m diameter, two bladed propeller is given in Fig. 30. It may be noticed that the torque is greater than zero at zero rotational speed, so the machine will start by itself. Once the machine is started, the torque curve looks much like the torque curve for the Darrieus in that the torque increases with increasing rotational speed until a peak torque is reached. The torque decreases rapidly past the peak until it reaches zero at the run away or free wheeling rotational speed.

A high solidity machine may have its maximum torque at zero speed. This is the case for the torque of the Savonius turbine shown in Fig. 31. This is the experimental torque curve for the machine shown in Fig. 1.4. The high torque at low speeds may be essential for some applications: for example, starting a larger Darrieus or operating a positive displacement pump.

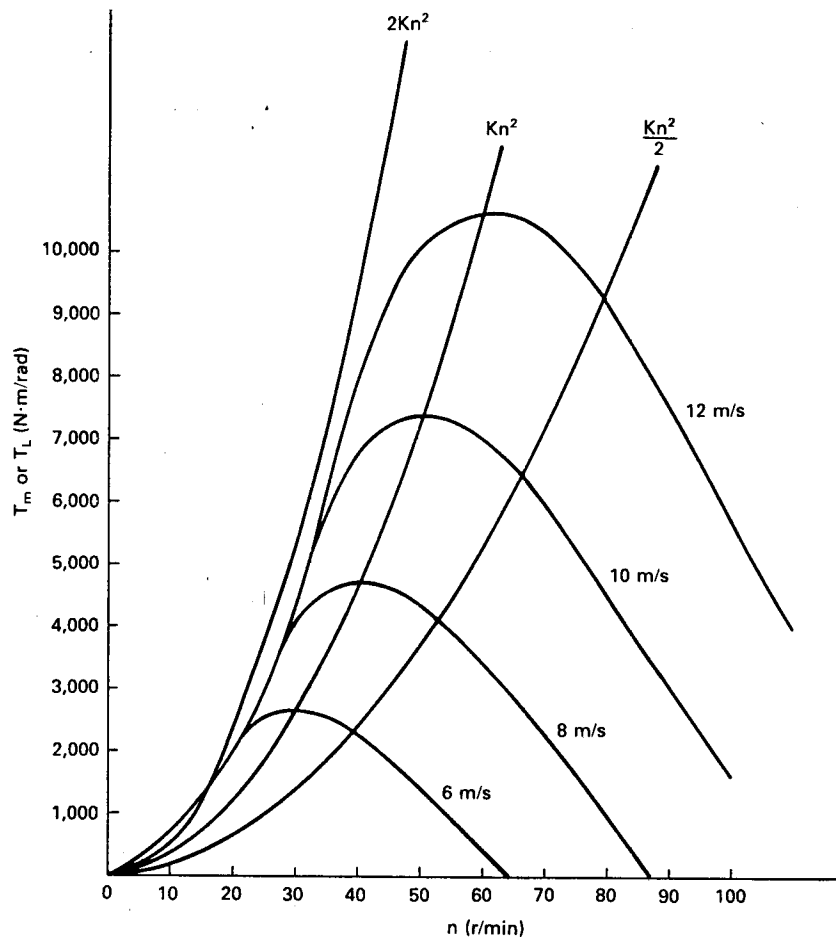


Figure 29: Three loads attached to Sandia 17-m turbine.

We have shown that variable speed turbine operation is technically possible. There is a possibility of simplified construction and lower costs and also a possibility of greater energy production from a given rotor. Fixed speed systems will probably dominate where induction or synchronous generators are used to supply power to a grid, but variable speed systems may find a role in stand alone situations as well as those situations where mechanical power is needed rather than electrical power.

10 PROBLEMS

1. A large turbine is rated at 2500 kW at standard conditions (0° and 101.3 kPa). What would be its rated power at the same rated wind speed if the temperature were 20°C and the turbine was located at 1500 m above sea level? Use the U. S. Standard Atmosphere

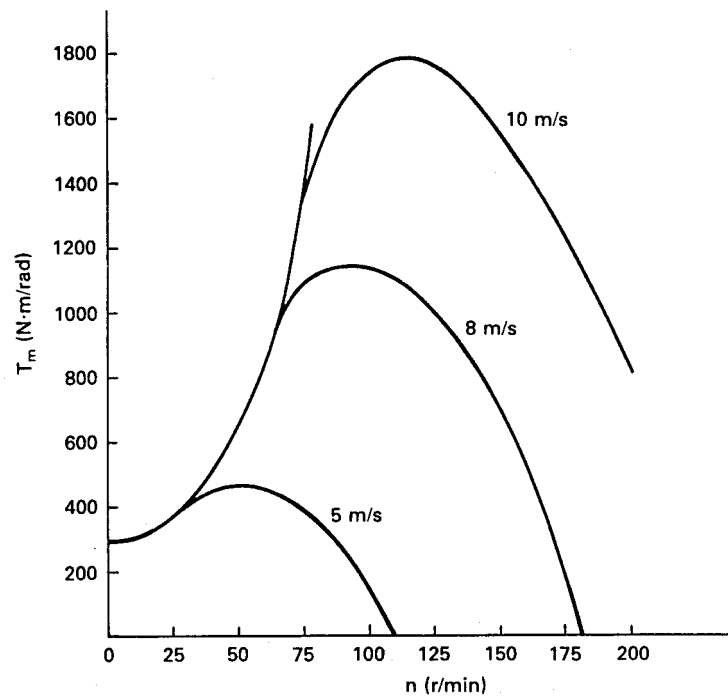


Figure 30: Torque versus rotational speed for low-solidity 10-m-diameter two-bladed propeller.

model discussed in Chapter 2 to find the average pressure.

2. An anemometer mounted on the nacelle of an operating downwind propeller type turbine measures an average wind speed of 10 m/s. Estimate the undisturbed wind speed, assuming this anemometer measures the same wind speed as seen by the propeller.
3. The Sandia 17-m turbine has a diameter of 16.7 m, an area of 187 m^2 , and is spinning at 42 r/min. The ambient temperature is 15°C and the pressure is 83 kPa. For each of the wind speeds 5, 7, 12, and 16 m/s,
 - (a) Find the tip speed ratio.
 - (b) Find the coefficient of performance from Fig. 9.
 - (c) Find the predicted mechanical power output. Compare these values with those given on Fig. 10.
4. An electric utility decides to add 50 MW of wind generation to its system. If the individual units are to be rated at 2 MW in a 13-m/s wind at standard conditions and have efficiencies $C_{pR} = 0.32$, $\eta_{mR} = 0.94$, and $\eta_{gR} = 0.96$, what is the required swept area A of each rotor? What is the rotor diameter, if the rotor is a two bladed horizontal axis propeller?

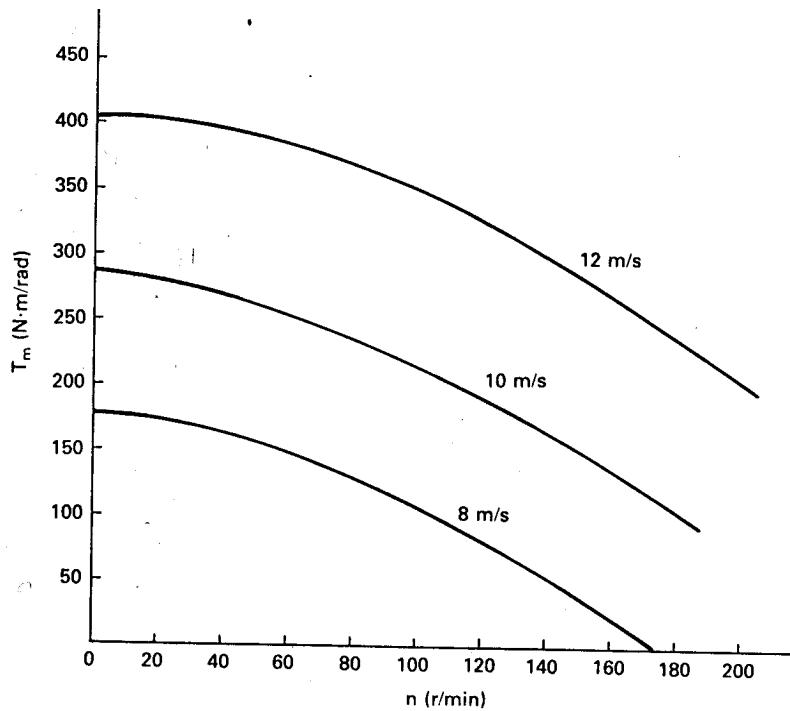


Figure 31: Torque versus rotational speed for Savonius turbine: radius 0.88 m; area 22 m².

5. The MOD-2 wind turbine is rated at 2500 kW at standard conditions in a 12.3 m/s wind speed. Assume a three stage gearbox with rated efficiency of 0.94 and a generator with rated efficiency 0.95, and estimate the output power in a 8 m/s wind speed. Include the effects of lowered efficiency due to lowered power transfer.
6. The NASA-DOE MOD-0 has a rated electrical output of 100 kW, a cut-in wind speed of 3.5 m/s, a rated wind speed of 8 m/s, a rotor diameter of 37.5 m, a fixed rotor rotational speed of 40 r/min, and a generator rotational speed of 1800 r/min.
 - (a) What is the tip speed ratio at rated wind speed?
 - (b) What is the tip speed ratio at cut-in?
 - (c) What is the overall efficiency of the turbine, drive train, and generator if $P_e = 100$ kW when the wind speed is 8 m/s? Assume standard conditions.
7. A MOD-2 wind turbine is delivering a mechanical power $P_m = 2000$ kW at 17.5 r/min to a gearbox with an output speed of 1800 r/min. The gearbox is 92 percent efficient at this power level.
 - (a) What is the average torque in the low speed shaft?

- (b) What is the average torque in the high speed shaft?
8. Assume that a wind turbine rated at 100 kW can be adequately modeled by the model turbine of Fig. 16, a rated wind speed of 7.7 m/s, a cut-in speed of 4.3 m/s, and a furling speed of 17.9 m/s. Determine the capacity factor and the monthly energy production in kWh (in a 30 day month) for sites where:
- (a) $c = 5.0$ and $k = 1.6$
 - (b) $c = 6.5$ and $k = 2.0$
 - (c) $c = 8.0$ and $k = 2.4$
9. A Sandia 17-m Darrieus is located at a site with wind characteristics $c = 7$ m/s and $k = 2.6$. Which configuration produces more energy each year, the 42 r/min machine rated at 25 kW in an 11 m/s wind speed, or the 52.5 r/min machine rated at 60 kW in a 15.5 m/s wind speed? Assume $u_c = 5$ m/s and 6.5 m/s respectively and that the furling speed is 20 m/s.
10. You are designing a wind turbine for which the cut-in wind speed is one-half the rated wind speed. What is the rated wind speed for which yearly energy production is maximized for
- (a) $c = 6$ m/s and $k = 1.4$
 - (b) $c = 6$ m/s and $k = 2.0$
 - (c) $c = 8$ m/s and $k = 2.6$
11. You are designing the low speed shaft for a horizontal axis turbine which has to transmit 50 kW of mechanical power at a rotational speed of 95 r/min. Solid steel shafts are available in half-inch increments starting at 2 inches outside diameter. The recommended maximum stress is 55 MPa. What size shaft should you specify?
12. An elastomeric shaft 2 m in length is used to deliver 100 kW of mechanical power to a generator at 1800 r/min. The shaft diameter is 0.05 m and the shear modulus G is 0.9 GPa.
- (a) What is the total twist θ in the shaft?
 - (b) What is the total energy stored in the shaft?
13. The generator of the previous problem is receiving 100 kW of mechanical power when it suddenly loses electrical load at time $t = 0$. The inertia I of the generator can be assumed to be $3.6 \text{ kg}\cdot\text{m}^2$.
- (a) Find an expression for θ as a function of time.
 - (b) How long does it take for θ to change $\pi/2$ radians?

- (c) A four pole synchronous generator changes the output electrical phase by π radians or 180° when the input mechanical position is changed by $\pi/2$ radians 90° . Is there any problem in reconnecting the generator to the electrical grid when it is in this position?
14. The Sandia 17-m Darrieus is to be operated at 52.5 r/min with the low speed shaft delivering an average power of 65 kW mechanical in winds of 15 m/s or more. The shaft is to be of solid steel with a recommended maximum stress of 55 MPa.
- (a) What approximate size of shaft is needed to transmit the average torque?
- (b) What approximate size shaft is needed to accept the peak torque at wind speeds up to about 20 m/s? You can estimate the peak torque from Fig. 20.
15. A wind turbine is being operated in a variable speed mode with an optimum load. If the wind speed doubles, what is the change in
- (a) output power?
- (b) torque?
- (c) rotor speed?
16. A Darrieus turbine produces a low-speed-shaft power of 531 kW at the rated speed of 30.8 r/min. It is started by applying rated torque of 164,500 N·m/rad for 13 seconds.
- (a) What is the moment of inertia?
- (b) If a constant torque clutch is used, how much energy does the clutch absorb during the start cycle?
- (c) If the steel used in the clutch plate has a specific heat capacity of 527 J/kg·°C and the average temperature rise in the plate can be no more than 100°C, what is the minimum mass of steel required in the plate?
17. A Darrieus turbine produces a low speed shaft power of 1650 kW at the rated speed of 22.1 r/min in a 14.3 m/s wind speed.
- (a) Determine rated rotor torque from the values given for measured power and rated rotor speed.
- (b) If the rotor is started at this rated torque in 18 seconds, determine the rotor inertia.
- (c) If a constant torque clutch is used, how much energy is absorbed during the start cycle?
- (d) If the generator is rated at 1800 r/min, what is the input torque to the generator, assuming no losses in the gearbox?

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