WIND CHARACTERISTICS

The wind blows to the south and goes round to the north:, round and round goes the wind, and on its circuits the wind returns. Ecclesiastes 1:6

The earth's atmosphere can be modeled as a gigantic heat engine. It extracts energy from one reservoir (the sun) and delivers heat to another reservoir at a lower temperature (space). In the process, work is done on the gases in the atmosphere and upon the earth-atmosphere boundary. There will be regions where the air pressure is temporarily higher or lower than average. This difference in air pressure causes atmospheric gases or wind to flow from the region of higher pressure to that of lower pressure. These regions are typically hundreds of kilometers in diameter.

Solar radiation, evaporation of water, cloud cover, and surface roughness all play important roles in determining the conditions of the atmosphere. The study of the interactions between these effects is a complex subject called meteorology, which is covered by many excellent textbooks.[4, 8, 20] Therefore only a brief introduction to that part of meteorology concerning the flow of wind will be given in this text.

1 METEOROLOGY OF WIND

The basic driving force of air movement is a difference in air pressure between two regions. This air pressure is described by several physical laws. One of these is Boyle's law, which states that the product of pressure and volume of a gas at a constant temperature must be a constant, or

$$p_1V_1 = p_2V_2 \tag{1}$$

Another law is Charles' law, which states that, for constant pressure, the volume of a gas varies directly with absolute temperature.

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \tag{2}$$

If a graph of volume versus temperature is made from measurements, it will be noticed that a zero volume state is predicted at -273.15^oC or 0 K.

The laws of Charles and Boyle can be combined into the *ideal gas law*

$$pV = nRT \tag{3}$$

In this equation, R is the universal gas constant, T is the temperature in kelvins, V is the volume of gas in m^3, n is the number of kilomoles of gas, and p is the pressure in pascals (N/m^2). At standard conditions, 0^oC and one atmosphere, one kilomole of gas occupies 22.414 m^3 and the universal gas constant is 8314.5 J/(kmol·K) where J represents a joule or a newton meter of energy. The pressure of one atmosphere at 0^oC is then

$$\frac{(8314.5\text{J}/(\text{kmol}\cdot\text{K}))(273.15\ \ \text{K})}{22.414\ \ \text{m}^3} = 101,325 \qquad \text{Pa} \tag{4}$$

One kilomole is the amount of substance containing the same number of molecules as there are atoms in 12 kg of the pure carbon nuclide ^{12}C. In dry air, 78.09 % of the molecules are nitrogen, 20.95 % are oxygen, 0.93 % are argon, and the other 0.03 % are a mixture of CO_2, Ne, Kr, Xe, He, and H_2. This composition gives an average molecular mass of 28.97, so the mass of one kilomole of dry air is 28.97 kg. For all ordinary purposes, dry air behaves like an ideal gas.

The density ρ of a gas is the mass m of one kilomole divided by the volume V of that kilomole.

$$\rho = \frac{m}{V} \tag{5}$$

The volume of one kilomole varies with pressure and temperature as specified by Eq. 3. When we insert Eq. 3 into Eq. 5, the density is given by

$$\rho = \frac{mp}{RT} = \frac{3.484p}{T} \qquad \text{kg/m}^3 \tag{6}$$

where p is in kPa and T is in kelvins. This expression yields a density for dry air at standard conditions of 1.293 kg/m^3.

The common unit of pressure used in the past for meteorological work has been the *bar* (100 kPa) and the *millibar* (100 Pa). In this notation a standard atmosphere was referred to as 1.01325 bar or 1013.25 millibar.

Atmospheric pressure has also been given by the height of mercury in an evacuated tube. This height is 29.92 inches or 760 millimeters of mercury for a standard atmosphere. These numbers may be useful in using instruments or reading literature of the pre-SI era. It may be worth noting here that several definitions of *standard conditions* are in use. The chemist uses 0^oC as standard temperature while engineers have often used 68^oF (20^oC) or 77^oF (25^oC) as standard temperature. We shall not debate the respective merits of the various choices, but note that some physical constants depend on the definition chosen, so that one must exercise care in looking for numbers in published tables. In this text, standard conditions will always be 0^oC and 101.3 kPa.

Within the atmosphere, there will be large regions of alternately high and low pressure.

These regions are formed by complex mechanisms, which are still not fully understood. Solar radiation, surface cooling, humidity, and the rotation of the earth all play important roles.

In order for a high pressure region to be maintained while air is *leaving* it at ground level, there must be air *entering* the region at the same time. The only source for this air is *above* the high pressure region. That is, air will flow *down* inside a high pressure region (and *up* inside a low pressure region) to maintain the pressure. This descending air will be warmed *adiabatically* (i.e. without heat or mass transfer to its surroundings) and will tend to become dry and clear. Inside the low pressure region, the rising air is cooled adiabatically, which may result in clouds and precipitation. This is why high pressure regions are usually associated with good weather and low pressure regions with bad weather.

A line drawn through points of equal pressure on a weather map is called an *isobar*. These pressures are corrected to a common elevation such as sea level. For ease of plotting, the intervals between the isobars are usually 300, 400, or 500 Pa. Thus, successive isobars would be drawn through points having readings of 100.0, 100.4, 100.8 kPa, etc. Such a map is shown in Fig. 1. This particular map of North America shows a low pressure region over the Great Lakes and a high pressure region over the Southwestern United States. There are two frontal systems, one in the Eastern United States and one in the Pacific Northwest. The map shows a range of pressures between 992 millibars (99.2 kPa) and 1036 millibars (103.6 kPa). These pressures are all corrected to sea level to allow a common basis for comparison. This means there are substantial areas of the Western United States where the actual measured station pressure is well below the value shown because of the station elevation above sea level.

The horizontal pressure difference provides the horizontal force or *pressure gradient* which determines the speed and initial direction of wind motion. In describing the direction of the wind, we always refer to the direction of origin of the wind. That is, a north wind is blowing on us *from* the north and is going *toward* the south.

The greater the pressure gradient, the greater is the force on the air, and the higher is the wind speed. Since the direction of the force is from higher to lower pressure, and perpendicular to the isobars, the initial tendency of the wind is to blow parallel to the horizontal pressure gradient and perpendicular to the isobars. However, as soon as wind motion is established, a deflective force is produced which alters the direction of motion. This force is called the *Coriolis force.*

The Coriolis force is due to the earth's rotation under a moving particle of air. From a fixed observation point in space air would appear to travel in a straight line, but from our vantage point on earth it appears to curve. To describe this change in observed direction, an equivalent force is postulated.

The basic effect is shown in Fig. 2. The two curved lines are lines of constant latitude, with point B located directly south of point A. A parcel of air (or some projectile like a cannon ball) is moving south at point A. If we can imagine our parcel of air or our cannon ball to have zero air friction, then the speed of the parcel of air will remain constant with respect to the ground. The direction will change, however, because of the earth's rotation under the parcel.

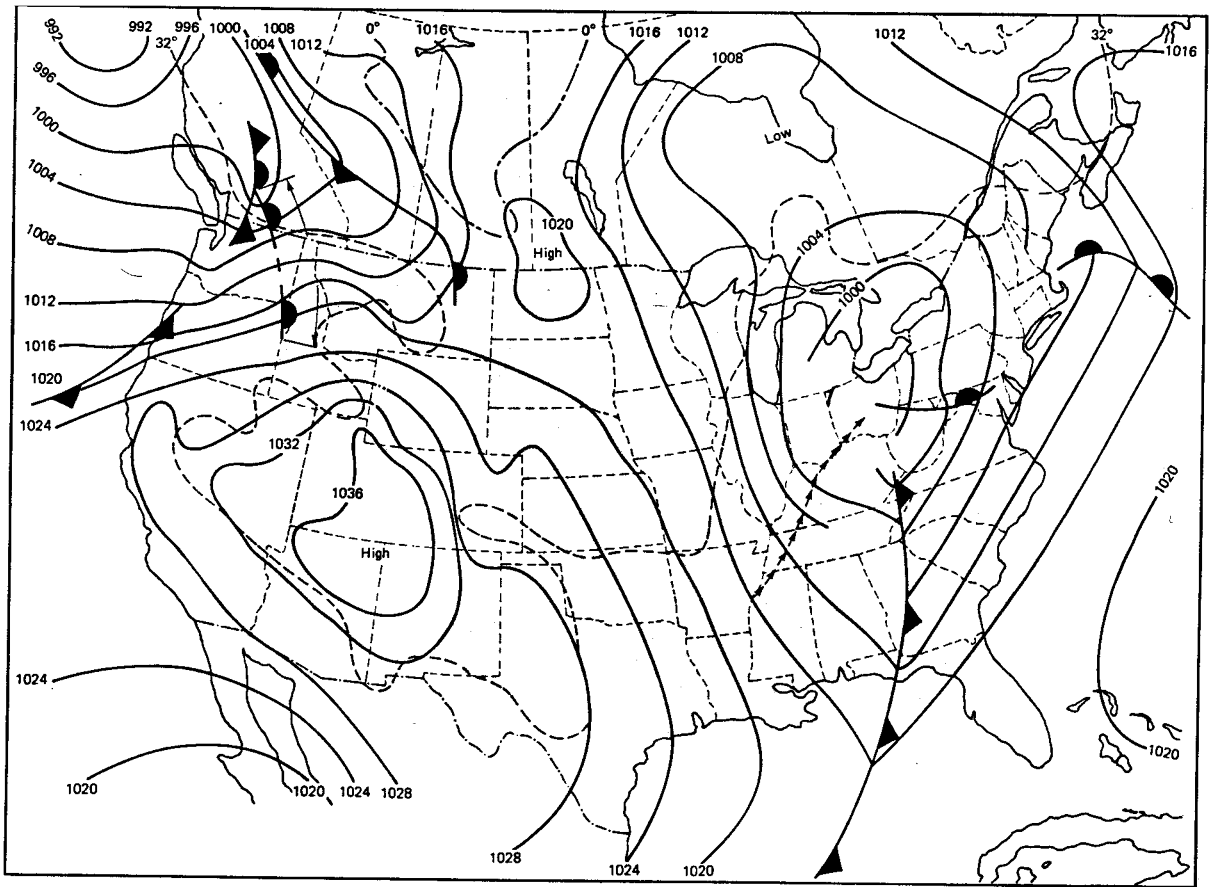

Figure 1: Weather map showing isobars

At point A, the parcel has the same eastward speed as the earth. Because of the assumed lack of friction, it will maintain this same eastward speed as it moves south. The eastward speed of the earth increases, however, as we move south (in the Northern Hemisphere). Therefore, the parcel will appear to have a westward component of velocity on the latitude line passing through point B. During the time required for the parcel to move from the first latitude line to the second, point A has moved eastward to point A' and point B has moved eastward to point B'. The path of the parcel is given by the dashed line. Instead of passing directly over point B' which is directly south of point A', the parcel has been deflected to the right and crosses the second latitude line to the west of B'. The total speed relative to the earth's surface remains the same, so the southward moving component has decreased to allow the westward moving component of speed to increase.

It can be shown that a parcel of air will deflect to its right in the Northern Hemisphere, regardless of the direction of travel. This is not an obvious truth, but the spherical geometry necessary to prove the statement is beyond the scope of this text. We shall therefore accept

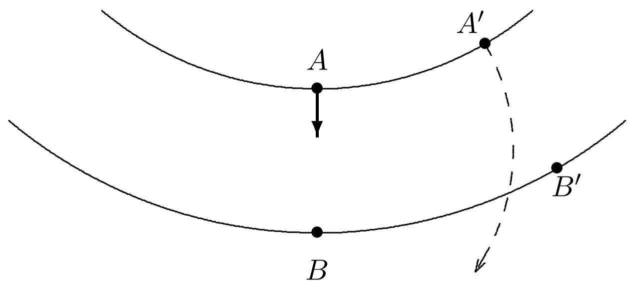

Figure 2: Coriolis force

it on faith.

Another statement we shall accept on faith is that the deflection of the parcel of air must cease when the wind direction becomes parallel to the isobars. Otherwise the wind would be blowing in the direction of increasing pressure, which would be like water running uphill. Since the Coriolis force acts in a direction 90 degrees to the right of the wind, it must act in a direction opposite to the pressure gradient at the time of maximum deflection. If there are no other forces present, this Coriolis force will exactly balance the pressure gradient force and the wind will flow parallel to the isobars, with higher pressure to the right of the wind direction. For straight or slightly curved isobars this resultant wind is called the *geostrophic wind.*

When strongly curved isobars are found, a *centrifugal force* must also be considered. Fig. 3 shows one isobar around a *cyclone*, which is a low pressure area rotating counterclockwise (Northern Hemisphere). Fig. 4 shows an isobar around a high pressure area which is rotating clockwise (Northern Hemisphere). This region is called an *anticyclone.* As mentioned earlier, the low pressure area is usually associated with bad weather, but does not imply anything about the magnitude of the wind speeds. A cyclone normally covers a major part of a state or several states and has rather gentle winds. It should not be confused with a tornado, which covers a very small region and has very destructive winds.

The wind moving counterclockwise in the cyclone experiences a pressure gradient force f_p inward, a Coriolis force f_c outward, and a centrifugal force f_g outward. For wind to continue moving in a counterclockwise direction parallel to the isobars, the forces must be balanced, so the pressure gradient force for a cyclone is

$$f_p = f_c + f_g \tag{7}$$

The pressure force inward is balanced by the sum of the Coriolis and centrifugal forces. The wind that flows in such a system is called the *gradient* wind. The term "geostrophic

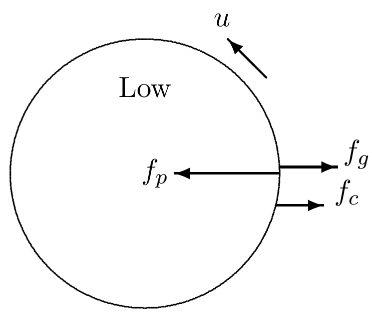

Figure 3: Wind forces in a low-pressure area

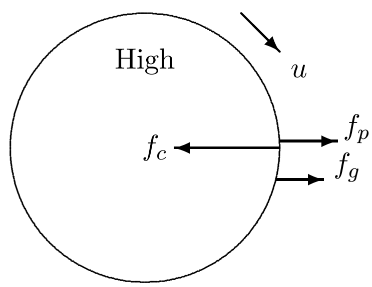

Figure 4: Wind forces in a high-pressure area

wind" applies only to a wind flowing in the vicinity of nearly straight isobars.

For the high pressure area of Fig. 4, the pressure and Coriolis forces reverse in direction. The pressure gradient force for an anticyclone is therefore

$$f_p = f_c - f_g \tag{8}$$

The difference between Eqs. 7 and 8 means that cyclones and anticyclones tend to stabilize at somewhat different relative pressures and wind speeds. Since the atmosphere is never completely stable, these differences are not usually of major concern.

2 WORLD DISTRIBUTION OF WIND

Ever since the days of sailing ships, it has been recognized that some areas of the earth's surface have higher wind speeds than others. Terms like *doldrums, horse latitudes*, and *trade winds* are well established in literature. A very general picture of prevailing winds over the surface of the earth is shown in Fig. 5. In some large areas or at some seasons, the actual pattern differs strongly from this idealized picture. These variations are due primarily to the irregular heating of the earth's surface in both time and position.

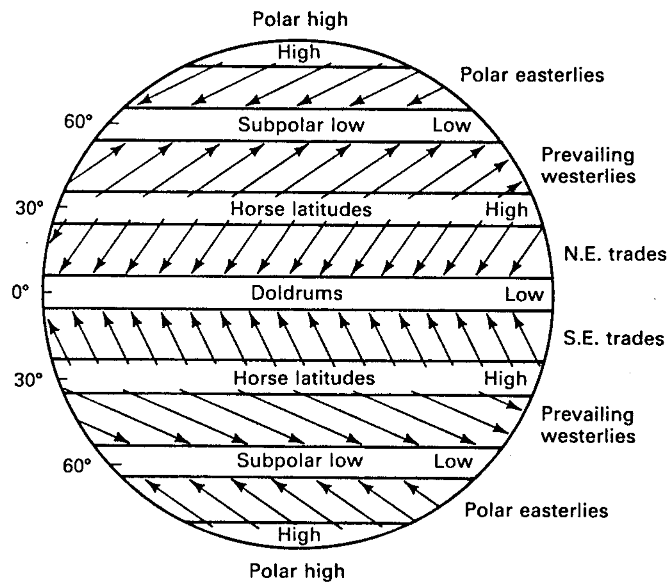

Figure 5: Ideal terrestrial pressure and wind systems

The *equatorial calms* or doldrums are due to a belt of low pressure which surrounds the earth in the equatorial zone as a result of the average overheating of the earth in this region. The warm air here rises in a strong convection flow. Late afternoon showers are common from the resulting adiabatic cooling which is most pronounced at the time of highest *diurnal* (daily) temperature. These showers keep the humidity very high without providing much surface cooling. The atmosphere tends to be oppressive, hot, and sticky with calm winds and slick glassy seas. Unless prominent land features change the weather patterns, regions near the equator will not be very good for wind power applications.

Ideally, there are two belts of high pressure and relatively light winds which occur symmetrically around the equator at 30^o N and 30^o S latitude. These are called the *subtropical calms* or *subtropical highs* or *horse latitudes*. The latter name apparently dates back to the sailing vessel days when horses were thrown overboard from becalmed ships to lighten the load and conserve water. The high pressure pattern is maintained by vertically descending air inside the pattern. This air is warmed adiabatically and therefore develops a low relative humidity with clear skies. The dryness of this descending air is responsible for the bulk of the world's great deserts which lie in the horse latitudes.

There are then two more belts of low pressure which occur at perhaps 60^o S latitude and 60^o N latitude, the *subpolar lows*. In the Southern Hemisphere, this low is fairly stable and does not change much from summer to winter. This would be expected because of the global encirclement by the southern oceans at these latitudes. In the Northern Hemisphere, however, there are large land masses and strong temperature differences between land and water. These cause the lows to reverse and become highs over land in the winter (the *Canadian* and *Siberian highs*). At the same time the lows over the oceans, called the *Iceland low* and the *Aleutian low*, become especially intense and stormy low pressure areas over the relatively warm North Atlantic and North Pacific Oceans.

Finally, the polar regions tend to be high pressure areas more than low pressure. The intensities and locations of these highs may vary widely, with the center of the high only rarely located at the geographic pole.

The combination of these high and low pressure areas with the Coriolis force produces the prevailing winds shown in Fig. 5. The *northeast* and *southeast trade winds* are among the most constant winds on earth, at least over the oceans. This causes some islands, such as Hawaii (20^o N. Latitude) and Puerto Rico (18^o N. Latitude), to have excellent wind resources. The westerlies are well defined over the Southern Hemisphere because of lack of land masses. Wind speeds are quite steady and strong during the year, with an average speed of 8 to 14 m/s. The wind speeds tend to increase with increasing southerly latitude, leading to the descriptive terms *roaring forties, furious fifties*, and *screaming sixties*. This means that islands in these latitudes, such as New Zealand, should be prime candidates for wind power sites.

In the Northern Hemisphere, the westerlies are quite variable and may be masked or completely reversed by more prominent circulation about moving low and high pressure areas. This is particularly true over the large land masses.

3 WIND SPEED DISTRIBUTION IN THE UNITED STATES

There are over 700 stations in the United States where meteorological data are recorded at regular intervals. Records at some stations go back over a century. Wind speeds are measured at these stations by devices called *anemometers*. Until recently, wind speed data were primarily recorded in either *knots* or *miles per hour* (mi/h), and any study of the old records will have to be done in these old units. A knot, short for nautical mile per hour, is equal to 0.5144 m/s. A mi/h is equal to 0.4470 m/s, which makes a knot equal to 1.15 mi/h.

Wind speed data are affected by the anemometer height, the exposure of the anemometer as regards the surrounding buildings, hills, and trees, the human factor in reading the wind speed, as well as the quality and maintenance of the anemometer. The standardization of these factors has not been well enforced over the years. For example, the standard anemometer height is 10 m but other heights are often found. In one study [16], six Kansas stations with data available from 1948 to 1976 had 21 different anemometer heights, none of which was 10 m. The only station which did not change anemometer height during this period was Russell, at a 9 m height. Heights ranged from 6 m to 22 m. The anemometer height at Topeka changed five times, with heights of 17.7, 22.3, 17.7, 22.0, and 7.6 m.

Reported wind data therefore need to be viewed with some caution. Only when anemometer heights and surrounding obstructions are the same can two sites be fairly compared for wind power potential. Reported data can be used, of course, to give an indication of the best regions of the country, with local site surveys being desirable to determine the quality of a potential wind power site.

Table 2.1 shows wind data for several stations in the United States. Similar data are available for most other U. S. stations. The percentage of time that the wind speed was recorded in a given speed range is given, as well as the mean speed and the peak speed[3]. Each range of wind speed is referred to as a *speed group*. Wind speeds were always recorded in integer knots or integer mi/h, hence the integer nature of the speed groups. If wind speeds were recorded in knots instead of the mi/h shown, the size of the speed groups would be adjusted so the percentage in each group remains the same. The speed groups for wind speeds in knots are 1-3, 4-6, 7-10, 11-16, 17-21, 22-27, 28-33, 34-40, and 41-47 knots. When converting wind data to mi/h speed groups, a 6 knot (6.91 mi/h) wind is assigned to the speed group containing 7 mi/h, while a 7 knot (8.06 mi/h) wind is assigned to the 8-12 mi/h speed group. Having the same percentages for two different unit systems is a help for some computations.

This table was prepared for the ten year period of 1951-60. No such tabulation was prepared by the U. S. Environmental Data and Information Service for 1961-70 or 1971-80, and no further tabulation is planned until it appears that the long term climate has changed enough to warrant the expense of such a compilation.

It may be seen that the mean wind speed varies by over a factor of two, from 3.0 m/s in Los Angeles to 7.8 m/s in Cold Bay, Alaska. The power in the wind is proportional to the cube of the wind speed, as we shall see in Chapter 4, so a wind of 7.8 m/s has 17.6 times the available power as a 3.0 m/s wind. This does not mean that Cold Bay is exactly 17.6 times as good as Los Angeles as a wind power site because of other factors to be considered later, but it does indicate a substantial difference.

The peak speed shown in Table 2.1 is the average speed of the fastest mile of air (1.6 km) to pass through a *run of wind* anemometer and is not the instantaneous peak speed of the peak gust, which will be higher. This type of anemometer will be discussed in Chapter 3.

The Environmental Data and Information Service also has wind speed data available on magnetic tape for many recording stations. This magnetic tape contains the complete record of a station over a period of up to ten years, including such items as temperature, air pressure, and humidity in addition to wind speed and direction. These data are typically recorded once

TABLE 2.1 Annual Percentage Frequency of Wind by Speed Groups and the Mean Speed[a]

Station	0–3 mi/h	4–7 mi/h	8–12 mi/h	13–18 mi/h	19–24 mi/h	25–31 mi/h	32–38 mi/h	Mean Speed (mi/h)	Peak Speed[b] (m/s)
Albuquerque	17	36	26	13	5	2		8.6	40.2
Amarillo	5	15	32	32	12	4	1	12.9	37.5
Boise	15	30	32	18	4	1		8.9	27.3
Boston	3	12	33	35	12	4	1	13.3	38.9
Buffalo	5	17	34	27	13	3	1	12.4	40.7
Casper	8	16	27	27	13	7	2	13.3	NA
Chicago(O'Hare)	8	22	33	27	8	2		11.2	38.9
Cleveland	7	18	35	29	9	2		11.6	34.9
Cold Bay	4	9	18	27	21	14	5	17.4	NA
Denver	11	27	34	22	5	2		10.0	29.0
Des Moines	3	17	38	29	10	3	1	12.1	34.0
Fargo	4	13	28	31	15	7	2	14.4	51.4
Ft. Worth	4	14	34	34	10	3		12.5	30.4
Great Falls	7	19	24	24	15	9	3	13.9	36.6
Honolulu	9	17	27	32	12	2		12.1	30.0
Kansas City	9	29	35	23	5	1		9.8	32.2
Los Angeles	28	33	27	11	1			6.8	21.9
Miami	14	20	34	20	2			8.8	59.0
Minneapolis	8	21	34	28	9	2		11.2	41.1
Oklahoma City	2	11	34	34	13	6	1	14.0	38.9
Topeka	11	19	30	27	10	2		11.2	36.2
Wake Island	1	6	27	48	17	2		14.6	NA
Wichita	4	12	30	31	16	5	1	13.7	44.7

[a]*Source: Climatography of the United States, Series 82:* Decennial Census of the United State Climate, "Summary of Hourly Observations, 1951–1960" (Table B).

[b]NA, not available

an hour but the magnetic tape only has data for every third hour, so there are eight wind speeds and eight wind directions per day available on the magnetic tape.

The available data can be summarized in other forms besides that of Table 2.1. One form is the *speed-duration curve* as shown in Fig. 6. The horizontal axis is in hours per year, with a maximum value of 8760 for a year with 365 days. The vertical axis gives the wind speed that is exceeded for the number of hours per year on the horizontal axis. For Dodge City, Kansas, for example, a wind speed of 4 m/s is exceeded 6500 hours per year while 10 m/s is exceeded 700 hours per year. For Kansas City, 4 m/s is exceeded 4460 hours and 10 m/s is exceeded only 35 hours a year. Dodge City is seen to be a better location for a wind turbine than Kansas City. Dodge City had a mean wind speed for this year of 5.68 m/s while Kansas City had a mean wind speed of 3.82 m/s. The anemometer height for both stations was 7 m above the ground.

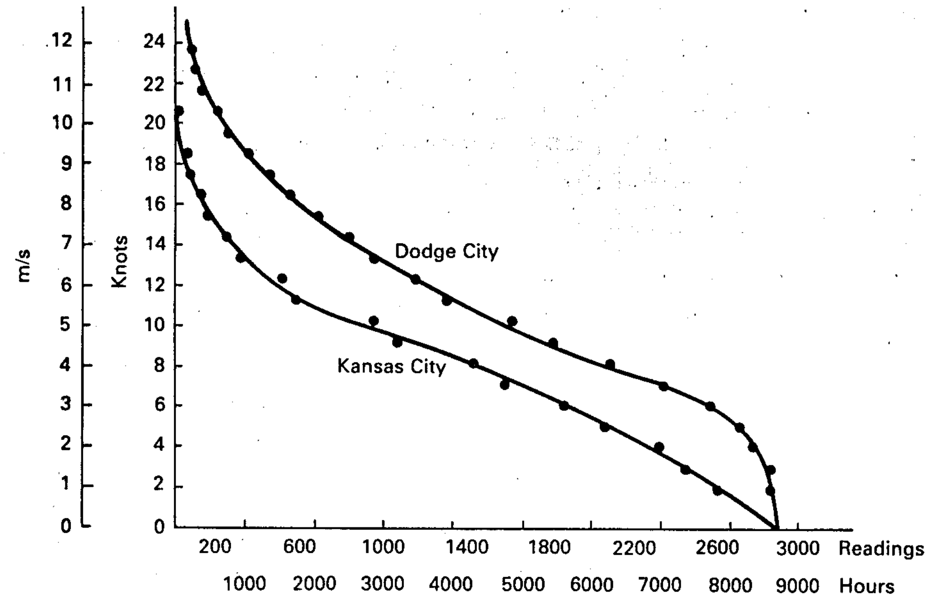

Figure 6: Speed-duration curves, 1970

Speed duration curves can be used to determine the number of hours of operation of a specific wind turbine. A wind turbine that starts producing power at 4 m/s and reaches rated power at 10 m/s would be operating 6500 hours per year at Dodge City (for the data shown in Fig. 6 and would be producing rated output for 700 of these hours. The output would be less than rated for the intermediate 5800 hours.

Speed duration curves do not lend themselves to many features of wind turbine design or selection. It is difficult to determine the optimum rated wind speed or the average power output from a speed duration curve, for example. For this reason, another type of curve has been developed, the *speed-frequency curve.* The two speed-frequency curves corresponding to the data of Fig. 6 are given in Fig. 7. These curves show the number of hours per year that the wind speed is in a given 1 m/s interval. At Dodge City the wind speed of 4 m/s is seen to occur 1240 hours per year. This actually means that we would expect wind speeds between 3.5 and 4.5 m/s for 1240 hours per year. The summation of the number of hours at each wind speed over all the wind speed intervals should be the total number of hours in the year.

Speed-frequency curves have several important features. One is that the intercept on the vertical axis is always greater than zero, due to the existence of calm spells at any site. Another feature is that the *most frequent speed* (the wind speed at the peak of this curve) is lower than the mean speed and varies with it. Still another feature is that the duration of the most frequent speed decreases as the mean speed increases[10].

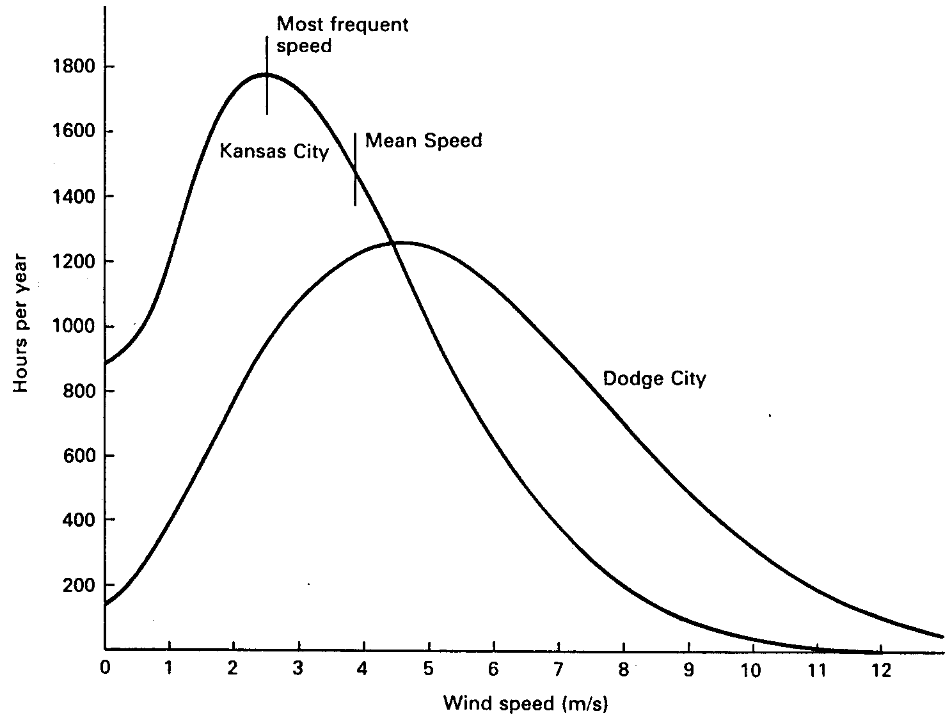

Figure 7: Speed-frequency curves, 1970

Speed frequency curves, or similar mathematical functions, can be used in studies to develop estimates of the seasonal and annual available wind power density in the United States and elsewhere. This is the power density in the wind in W/m^2 of area perpendicular to the flow of air. It is always substantially more than the power density that can actually be extracted from the wind, as we shall see in Chapter 4. The result of one such study[9] is shown in Fig. 8. This shows the estimated annual average power density available in the wind in W/m^2 at an elevation of 50 m above ground. Few wind data are recorded at that height, so the wind speeds at the actual anemometer heights have been extrapolated to 50 m by using the one-seventh power law, which will be discussed later in this chapter.

The shaded areas indicate mountainous terrain in which the wind power density estimates represent lower limits for exposed ridges and mountain tops. (From [9])

The map shows that the good wind regions are the *High Plains* (a north-south strip about 500 km wide on the east side of the Rocky Mountains), and mountain tops and exposed ridges throughout the country. The coastal regions in both the northeastern and northwestern United States are also good. There is a definite trend toward higher wind power densities at higher latitudes, as would be expected from Fig. 5. The southeastern United States is seen to be quite low compared with the remainder of the country.

There are selected sites, of course, which cannot be shown on this map scale, but which

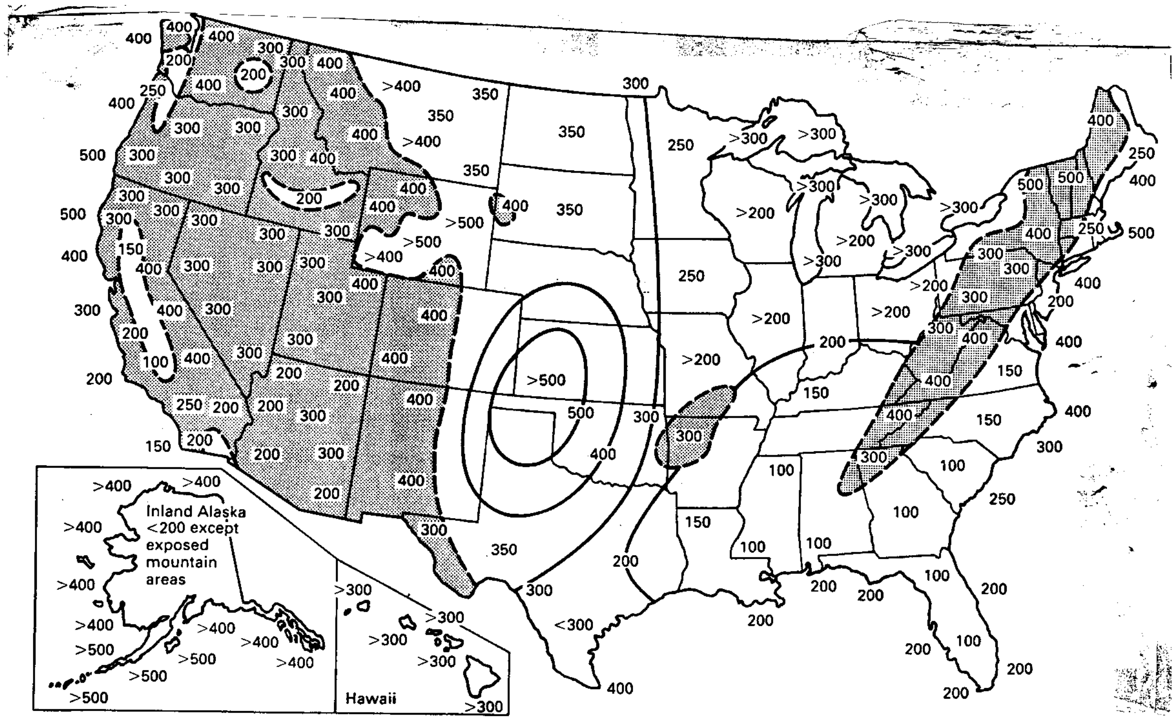

Figure 8: Annual mean wind power density W/m^2 estimated at 50 m above exposed areas.

have much higher wind power densities. Mt. Washington in New Hampshire has an average wind speed of over 15 m/s, twice that of the best site in Table 2.1. The annual average wind power density there would be over 3000 W/m^2. Mt. Washington also has the distinction of having experienced the highest wind speed recorded at a regular weather data station, 105 m/s (234 mi/h). Extreme winds plus severe icing conditions make this particular site a real challenge for the wind turbine designer. Other less severe sites are being developed first for these reasons.

Superior sites include mountain passes as well as mountain peaks. When a high or low pressure air mass arrives at a mountain barrier, the air is forced to flow either over the mountain tops or through the passes. A substantial portion flows through the passes, with resulting high speed winds. The mountain passes are also usually more accessible than mountain peaks for construction and maintenance of wind turbines. One should examine each potential site carefully in order to assure that it has good wind characteristics before installing any wind turbines. Several sites should be investigated if possible and both hills and valleys should be considered.

4 ATMOSPHERIC STABILITY

As we have mentioned, most wind speed measurements are made about 10 m above the ground. Small wind turbines are typically mounted 20 to 30 m above ground level, while the propeller tip may reach a height of more than 100 m on the large turbines, so an estimate of wind speed variation with height is needed. The mathematical form of this variation will be considered in the next section, but first we need to examine a property of the atmosphere which controls this vertical variation of wind speed. This property is called *atmospheric stability*, which refers to the amount of mixing present in the atmosphere. We start this discussion by examining the pressure variation with height in the lower atmosphere

A given parcel of air has mass and is attracted to the earth by the force of gravity. For the parcel not to fall to the earth's surface, there must be an equal and opposite force directed away from the earth. This force comes from the decrease in air pressure with increasing height. The greater the density of air, the more rapidly must pressure decrease upward to hold the air at constant height against the force of gravity. Therefore, pressure decreases quickly with height at low altitudes, where density is high, and slowly at high altitudes where density is low. This balanced force condition is called *hydrostatic balance* or *hydrostatic equilibrium*.

The average atmospheric pressure as a function of elevation above sea level for middle latitudes is shown in Fig. 9. This curve is part of the model for the U.S. Standard Atmosphere. At sea level and a temperature of 273 K, the average pressure is 101.3 kPa, as mentioned earlier. A pressure of half this value is reached at about 5500 m. This pressure change with elevation is especially noticeable when flying in an airplane, when one's ears tend to 'pop' as the airplane changes altitude.

It should be noticed that the independent variable z is plotted on the vertical axis in this figure, while the dependent variable is plotted along the horizontal axis. It is plotted this way because elevation is intuitively up. To read the graph when the average pressure at a given height is desired, just enter the graph at the specified height, proceed horizontally until you hit the curve, and then go vertically downward to read the value of pressure.

The pressure in Fig. 9 is assumed to not vary with local temperature. That is, it is assumed that the column of air directly above the point of interest maintains the same mass throughout any temperature cycle. A given mass of air above some point will produce a constant pressure regardless of the temperature. The volume of gas in the column will change with temperature but not the pressure exerted by the column. This is not a perfect assumption because, while the mass of the entire atmosphere does not vary with temperature, the mass directly overhead will vary somewhat with temperature. A temperature decrease of 30^oC will often be associated with a pressure increase of 2 to 3 kPa. The atmospheric pressure tends to be a little higher in the early morning than in the middle of the afternoon. Winter pressures tend to be higher than summer pressures. This effect is smaller than the pressure variation due to movement of weather patterns, hence will be ignored in this text.

It will be seen later that the power output of a wind turbine is proportional to air density, which in turn is proportional to air pressure. A given wind speed therefore produces less power from a particular turbine at higher elevations, because the air pressure is less. A wind turbine located at an elevation of 1000 m above sea level will produce only about 90 % of the power it would produce at sea level, for the same wind speed and air temperature. There are many good wind sites in the United States at elevations above 1000 m, as can be seen by comparing Fig. 8 with a topographical map of the United States. Therefore this pressure variation with elevation must be considered in both technical and economic studies of wind power.

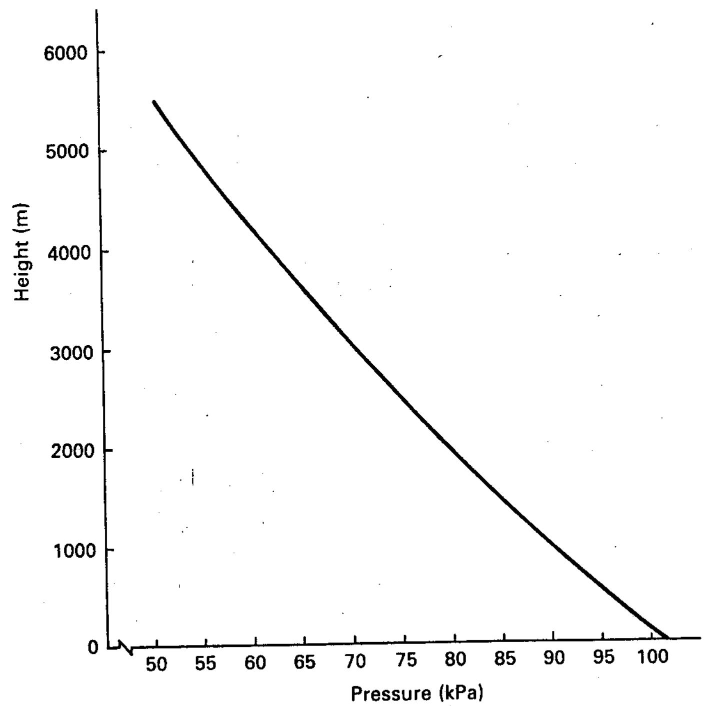

Figure 9: Pressure variation with altitude for U.S. Standard Atmosphere

The air density at a proposed wind turbine site is estimated by finding the average pressure at that elevation from Fig. 9 and then using Eq. 6 to find density. The ambient temperature must be used in this equation.

Example

A wind turbine is rated at 100 kW in a 10 m/s wind speed in air at standard conditions. If power output is directly proportional to air density, what is the power output of the turbine in a 10 m/s wind speed at Medicine Bow, Wyoming (elevation 2000 m above sea level) at a temperature of 20^oC?

From Fig. 9, we read an average pressure of 79.4 kPa. The density at $20^o\text{C} = 293$ K is then

$$\rho = \frac{3.484(79.4)}{293} = 0.944$$

The power output at these conditions is just the ratio of this density to the density at standard conditions times the power at standard conditions.

$$P_{\rm new} = P_{\rm old}\frac{\rho_{\rm new}}{\rho_{\rm old}} = 100\left(\frac{0.944}{1.293}\right) = 73 \ \ \text{kW}$$

The power output has dropped from 100 kW to 73 kW at the same wind speed, just because we have a smaller air density.

We have seen that the average pressure at a given site is a function of elevation above sea level. Ground level temperatures vary in a minor way with elevation, but are dominated by latitude and topographic features. Denver, Colorado, with an elevation of 1610 m above sea level has a slightly higher winter temperature average than Topeka, Kansas with an elevation of 275 m, for example. We must be cautious, therefore, about estimating ground level temperatures based on elevation above sea level.

Once we leave the ground, however, and enter the first few thousand meters of the atmosphere, we find a more predictable temperature decrease with altitude. We shall use the word *altitude* to refer to the height of an object above ground level, and *elevation* to refer to the height above sea level. With these definitions, a pilot flying in the mountains is much more concerned about his altitude than his elevation.

This temperature variation with altitude is very important to the character of the winds in the first 200 m above the earth's surface, so we shall examine it in some detail. We observe that Eq. 3 (the ideal gas law or the *equation of state*) can be satisfied by pressure and temperature decreasing with altitude while the volume of one kmol (the *specific volume*) is increasing. Pressure and temperature are easily measured while specific volume is not. It is therefore common to use the *first law of thermodynamics* (which states that energy is conserved) and eliminate the volume term from Eq. 3. When this is done for an adiabatic process , the result is

$$\frac{T_1}{T_o} = \left(\frac{p_1}{p_o}\right)^{R/c_p} \tag{9}$$

where T_1 and p_1 are the temperature and pressure at state 1, T_o and p_o are the temperature and pressure at state 0, R is the universal gas constant, and c_p is the *constant-pressure specific heat* of air. The derivation of this equation may be found in most introductory thermodynamics books. The average value for the ratio R/c_p in the lower atmosphere is 0.286.

Eq. 9, sometimes called *Poisson's equation*, relates adiabatic temperature changes experienced by a parcel of air undergoing vertical displacement to the pressure field through which it moves. If we know the initial conditions T_o and p_o, we can calculate the temperature T_1 at any pressure p_1 as long as the process is adiabatic and involves only ideal gases.

Ideal gases can contain no liquid or solid material and still be ideal. Air behaves like an ideal gas as long as the water vapor in it is not saturated. When saturation occurs, water starts to condense, and in condensing gives up its *latent heat of vaporization*. This heat energy input violates the adiabatic constraint, while the presence of liquid water keeps air from being an ideal gas.

Example

A parcel of air at sea level undergoes an upward displacement of 2 km. The initial temperature is 20^oC. For the standard atmosphere of Fig. 9 and an adiabatic process, what is the temperature of the parcel at 2 km?

From Fig. 9 the pressure at 2 km is $p_1 = 79.4$ kPa. The temperature is, from Eq. 9,

$$T_1 = T_o \left(\frac{p_1}{p_o}\right)^{0.286} = 293 \left(\frac{79.4}{101.3}\right)^{0.286} = 273.3\text{K}$$

We see from this example that a parcel of air which undergoes an upward displacement experiences a temperature decrease of about 1^oC/100 m in an adiabatic process. This quantity is referred to as the *adiabatic lapse rate*, the *dry- adiabatic lapse rate*, or the *temperature gradient*. It is an ideal quantity, and does not vary with actual atmospheric parameters.

This ideal temperature decrease is reasonably linear up to several kilometers above the earth's surface and can be approximated by

$$T_a(z) = T_g - R_a(z - z_g) \tag{10}$$

where $T_a(z)$ is the temperature at elevation z m above sea level if all processes are adiabatic, T_g is the temperature at ground level z_g, and R_a is the adiabatic lapse rate, 0.01^oC/m. The quantity $z - z_g$ is the altitude above ground level.

The actual temperature decrease with height will normally be different from the adiabatic prediction, due to mechanical mixing of the atmosphere. The actual temperature may decrease more rapidly or less rapidly than the adiabatic decrease. In fact, the actual temperature can even increase for some vertical intervals. A plot of some commonly observed temperature variations with height is shown in Fig. 10. We shall start the explanation at 3 p.m., at which time the earth has normally reached its maximum temperature and the first 1000 m or so above the earth's surface is well mixed. This means that the curve of actual temperature will follow the theoretical adiabatic curve rather closely.

By 6 p.m. the ground temperature has normally dropped slightly. The earth is much more effective at receiving energy from the sun and reradiating it into space than is the air above the earth. This means that the air above the earth is cooled and heated by conduction and convection from the earth, hence the air temperature tends to lag behind the ground temperature. This is shown in Fig. 10b, where at 6 p.m. the air temperature is slightly above the adiabatic line starting from the current ground temperature. As ground temperature continues to drop during the night, the difference between the actual or prevailing temperature and the adiabatic curve becomes even more pronounced. There may even be a *temperature*

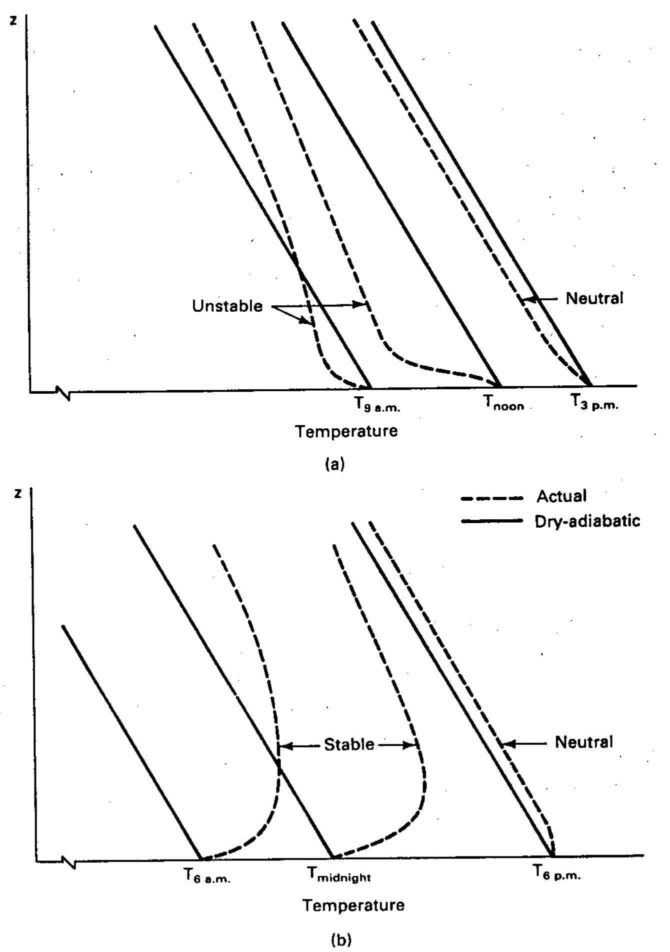

Figure 10: Dry-adiabatic and actual temperature variations with height as a function of time of day, for clear skies.

inversion near the ground. This usually occurs only with clear skies, which allow the earth to radiate its energy into space effectively.

When sunlight strikes the earth in the morning, the earth's surface temperature rises rapidly, producing the air temperature variations shown in Fig. 10a. The difference between the actual air temperature and the adiabatic curve would normally reach its maximum before noon, causing a relatively rapid heating of the air. There will be a strong convective flow of air during this time because a parcel of air that is displaced upward will find itself warmer and hence lighter than its surroundings. It is then accelerated upwards under hydrostatic pressure. It will continue to rise until its temperature is the same as that of the adiabatic curve. Another parcel has to move down to take the first parcel's place, perhaps coming down where the earth is not as effective as a collector of solar energy, and hence causes the atmosphere to be well mixed under these conditions. This condition is referred to as an *unstable atmosphere*.

On clear nights, however, the earth will be colder than the air above it, so a parcel at the temperature of the earth that is displaced upward will find itself colder than the surrounding air. This makes it more dense than its surroundings so that it tends to sink back down to its original position. This condition is referred to as a *stable atmosphere*. Atmospheres which have temperature profiles between those for unstable and stable atmospheres are referred to as *neutral atmospheres*. The daily variation in atmospheric stability and surface wind speeds is called the *diurnal cycle*.

It is occasionally convenient to express the actual temperature variation in an equation similar to Eq. 10. Over at least a narrow range of heights, the prevailing temperature $T_p(z)$ can be written as

$$T_p(z) = T_g - R_p(z - z_g) \tag{11}$$

where R_p is the slope of a straight line approximation to the actual temperature curve called the *prevailing lapse rate*. T_g is the temperature at ground level, z_g m above mean sea level, and z is the elevation of the observation point above sea level. We can force this equation to fit one of the dashed curves of Fig. 10 by using a *least squares* technique, and determine an approximate *lapse rate* for that particular time. If we do this for all times of the day and all seasons of the year, we find that the average prevailing lapse rate R_p is 0.0065^{o}C/m.

Suppose that a parcel of air is heated above the temperature of the neighboring air so it is now buoyant and will start to rise. If the prevailing lapse rate is less than adiabatic, the parcel will rise until its temperature is the same as the surrounding air. The pressure force and hence the acceleration of the parcel is zero at the point where the two lapse rate lines intersect. The upward velocity, however, produced by acceleration from the ground to the height at which the buoyancy vanishes, is greatest at that point. Hence the air will continue upward, now colder and more dense than its surroundings, and decelerate. Soon the upward motion will cease and the parcel will start to sink. After a few oscillations about that level the parcel will settle near that height as it is slowed down by friction with the surrounding air.

Example

Suppose that the prevailing lapse rate is 0.0065^{o}C/m and that a parcel of air is heated to 25^{o}C while the surrounding air at ground level is at 24^{o}C. Ground level is at an elevation 300 m above sea level. What will be the final altitude of the heated parcel after oscillations cease, assuming an adiabatic process?

The temperature variation with height for the linear adiabatic lapse rate is, from Eq. 10,

$$T_a = 25 - 0.01(z - 300)$$

Similarly, the temperature variation for the prevailing lapse rate is, from Eq. 11,

$$T_p = 24 - 0.0065(z - 300)$$

The two temperatures are the same at the point of equal buoyancy. Setting T_a equal to T_p and solving for z yields an intersection height of about 585 m above sea level or 285 m above ground level. This is illustrated in Fig. 11.

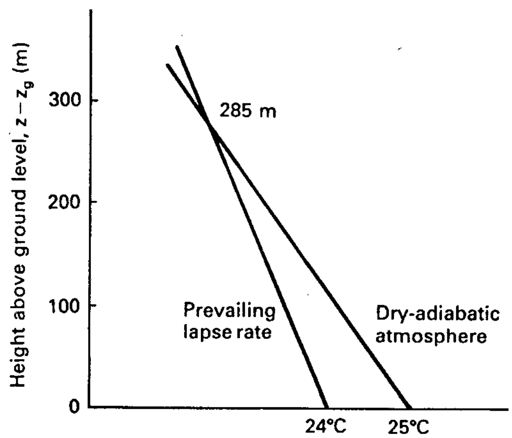

Figure 11: Buoyancy of air in a stable atmosphere

Atmospheric mixing may be limited to a relatively shallow layer if there is a temperature inversion at the top of the layer. This situation is illustrated in Fig. 12. Heated parcels rise until their temperature is the same as that of the ambient air, as before. Now, however, a doubling of the initial temperature difference does not result in a doubling of the height of the intersection, but rather yields a minor increase. Temperature inversions act as very strong lids against penetration of air from below, trapping the surface air layer underneath the inversion base. Such a situation is responsible for the maintenance of smog in the Los Angeles basin.

A more detailed system of defining stability is especially important in atmospheric pollution studies. Various stability parameters or categories have been defined and are available in the literature[18].

A stable atmosphere may have abrupt changes in wind speed at a boundary layer. The winds may be nearly calm up to an elevation of 50 or 100 m, and may be 20 m/s above that boundary. A horizontal axis wind turbine which happened to have its hub at this boundary would experience very strong *bending moments* on its blades and may have to be shut down in such an environment. An unstable atmosphere will be better mixed and will not evidence such sharp boundaries.

The wind associated with an unstable atmosphere will tend to be gusty, because of thermal mixing. Wind speeds will be quite small near the ground because of ground friction, increasing upward for perhaps several hundred meters over flat terrain. A parcel of heated air therefore leaves the ground with a low horizontal wind speed. As it travels upward, the surrounding air exerts drag forces on the parcel, tending to speed it up. The parcel will still maintain its identity for some time, traveling slower than the surrounding air.

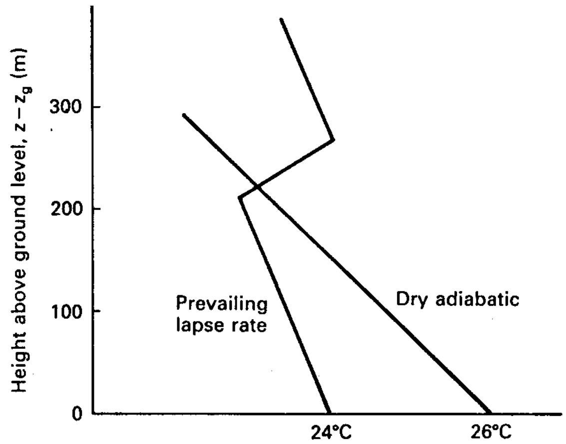

Figure 12: Deep buoyant layer topped by temperature inversion.

Parcels of air descending from above have higher horizontal speeds. They mix with the ascending parcels, causing an observer near the ground to sense wind speeds both below and above the mean wind speed. A parcel with higher velocity is called a *gust*, while a parcel with lower velocity is sometimes called a *negative gust* or a *lull*. These parcels vary widely in size and will hit a particular point in a random fashion. They are easily observed in the wave structure of a lake or in a field of tall wheat.

Gusts pose a hazard to large wind turbines because of the sudden change in wind speed and direction experienced by the turbine during a gust. Vibrations may be established or structural damage done. The wind turbine must be designed to withstand the peak gust that it is likely to experience during its projected lifetime.

Gusts also pose a problem in the adjusting of the turbine blades during operation in that the sensing anemometer will experience a different wind speed from that experienced by the blades. A gust may hit the anemometer and cause the blade *angle of attack* (the angle at which the blade passes through the air) to be adjusted for a higher wind speed than the blades are actually experiencing. This will generally cause the power output to drop below the optimum amount. Conversely, when the sensing anemometer experiences a lull, the turbine may be *trimmed* (adjusted) to produce more than rated output because of the higher wind speed it is experiencing. These undesirable situations require the turbine control system to be rather complex, and to have long time constants. This means that the turbine will be aimed slightly off the instantaneous wind direction and not be optimally adjusted for the instantaneous wind speed a large fraction of the time. The power production of the turbine will therefore be somewhat lower than would be predicted for an optimally adjusted and aimed wind turbine in a steady wind. The amount of this reduction is difficult to measure or even to estimate, but could easily be on the order of a 10 % reduction. This makes the simpler vertical axis machines more competitive than they might appear from wind tunnel tests since they require no aiming or blade control.

Thus far, we have talked only about gusts due to *thermal turbulence*, which has a very strong diurnal cycle. Gusts are also produced by *mechanical turbulence*, caused by higher speed winds flowing over rough surfaces. When strong frontal systems pass through a region, the atmosphere will be mechanically mixed and little or no diurnal variation of wind speed will be observed. When there is no significant mixing of the atmosphere due to either thermal or mechanical turbulence, a boundary layer may develop with relatively high speed laminar flow of air above the boundary and essentially calm conditions below it. Without the effects of mixing, upper level wind speeds tend to be higher than when mixed with lower level winds. Thus it is quite possible for nighttime wind speeds to decrease near the ground and increase a few hundred meters above the ground. This phenomenon is called the *nocturnal jet*. As mentioned earlier, most National Weather Service (NWS) anemometers have been located about 10 m above the ground, so the height and frequency of occurrence of this nocturnal jet have not been well documented at many sites. Investigation of nocturnal jets is done either with very tall towers (e.g. 200 m) or with meteorological balloons. Balloon data are not very precise, as we shall see in more detail in the next chapter, but rather long term records are available of National Weather Service balloon launchings. When used with appropriate caution, these data can show some very interesting variations of wind speed with height and time of day.

Fig. 13 shows the diurnal wind speed pattern at Dodge City, Kansas for the four year period, 1970-73, as recorded by the National Weather Service with an anemometer at 7 m above the ground. The average windspeed for this period was 5.66 m/s at this height. The spring season (March, April, and May) is seen to have the highest winds, with the summer slightly lower than the fall and winter seasons. The wind speed at 7 m is seen to have its peak during the middle of the day for all seasons.

Also shown in Fig. 13 are the results from two balloon launchings per day for the same period for three Kansas locations, Dodge City, Goodland, and Wichita. The terrain and wind characteristics are quite similar at each location, and averaging reduces the concern about missing data or local anomalies. The surface wind speed is measured at the time of the launch. These values lie very close to the Dodge City curves, indicating reasonable consistency of data. The wind speed pattern indicated by the balloons at 216 m is then seen to be much different from the surface speeds. The diurnal cycle at 216 m is opposite that at the surface. The lowest readings occur at noon and the highest readings occur at midnight. The lowest readings at 216 m are higher than the highest surface winds by perhaps 10-20 %, while the midnight wind speeds at 216 m are double those on the surface. This means that a wind turbine located in the nocturnal jet will have good winds in the middle of the day and even better winds at night. The average wind speed at 216 m for this period was 9.22 m/s, a very respectable value for wind turbine operation.

Measurements at intermediate heights will indicate some height where there is no diurnal cycle. A site at which the wind speed averages 7 or 8 m/s is a good site, especially if it is relatively steady. This again indicates the importance of detailed wind measurements at any

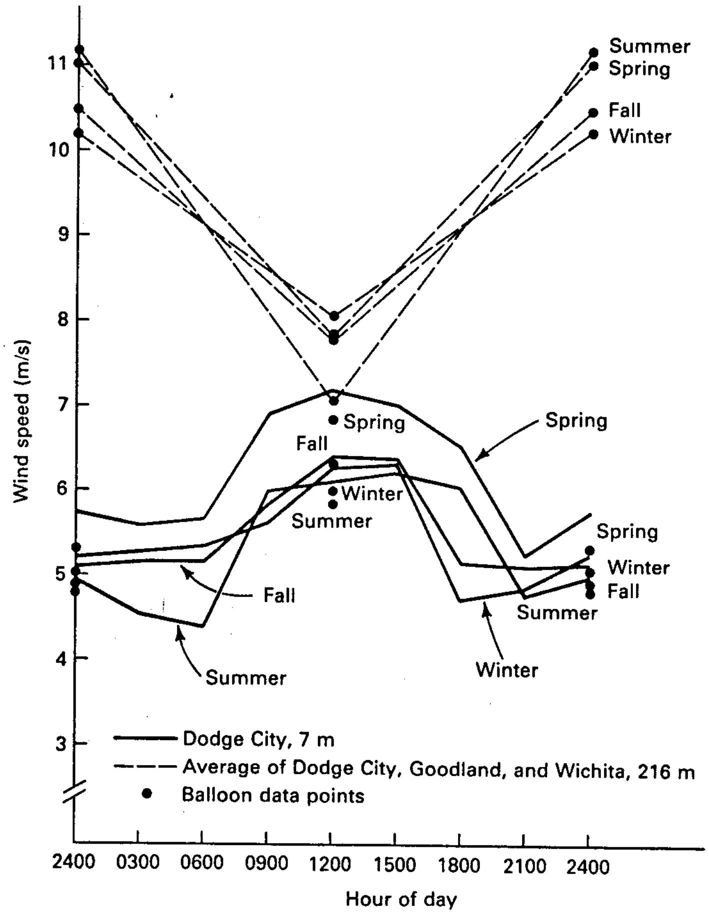

Figure 13: Western Kansas wind data, 1970-1973.

proposed site. Two very similar sites may have the same surface winds, but one may have a well developed nocturnal jet at 20 m while the other may never have a nocturnal jet below 200 m. The energy production of a wind turbine on a tall tower at the first site may be nearly double that at the second site, with a corresponding decrease in the cost of electricity. We see that measurements really need to be taken at heights up to the hub height plus blade radius over a period of at least a year to clearly indicate the actual available wind.

5 WIND SPEED VARIATION WITH HEIGHT

As we have seen, a knowledge of wind speeds at heights of 20 to 120 m above ground is very desirable in any decision about location and type of wind turbine to be installed. Many times, these data are not available and some estimate must be made from wind speeds measured at about 10 m. This requires an equation which predicts the wind speed at one height in terms of the measured speed at another, lower, height. Such equations are developed in texts on fluid mechanics. The derivations are beyond the scope of this text so we shall just use the results. One possible form for the variation of wind speed $u(z)$ with height z is

$$u(z) = \frac{u_f}{K}\left[\ln\frac{z}{z_o} - \xi\left(\frac{z}{L}\right)\right] \tag{12}$$

Here u_f is the *friction velocity*, K is the *von Kármán's constant* (normally assumed to be 0.4), z_o is the surface roughness length, and L is a scale factor called the *Monin Obukov length*[17, 13]. The function $\xi(z/L)$ is determined by the net solar radiation at the site. This equation applies to short term (e.g. 1 minute) average wind speeds, but not to monthly or yearly averages.

The surface roughness length z_o will depend on both the size and the spacing of roughness elements such as grass, crops, buildings, etc. Typical values of z_o are about 0.01 cm for water or snow surfaces, 1 cm for short grass, 25 cm for tall grass or crops, and 1 to 4 m for forest and city[19]. In practice, z_o cannot be determined precisely from the appearance of a site but is determined from measurements of the wind. The same is true for the friction velocity u_f, which is a function of surface friction and the air density, and $\xi(z/L)$. The parameters are found by measuring the wind at three heights, writing Eq. 12 as three equations (one for each height), and solving for the three unknowns u_f, z_o, and $\xi(z/L)$. This is not a linear equation so nonlinear analysis must be used. The results must be classified by the direction of the wind and the time of year because z_o varies with the upwind surface roughness and the condition of the vegetation. Results must also be classified by the amount of net radiation so the appropriate functional form of $\xi(z/L)$ can be used.

All of this is quite satisfying for detailed studies on certain critical sites, but is too difficult to use for general engineering studies. This has led many people to look for simpler expressions which will yield satisfactory results even if they are not theoretically exact. The most common of these simpler expressions is the power law, expressed as

$$\frac{u(z_2)}{u(z_1)} = \left(\frac{z_2}{z_1}\right)^\alpha \tag{13}$$

In this equation z_1 is usually taken as the height of measurement, approximately 10 m, and z_2 is the height at which a wind speed estimate is desired. The parameter α is determined empirically. The equation can be made to fit observed wind data reasonably well over the range of 10 to perhaps 100 or 150 m if there are no sharp boundaries in the flow.

The exponent α varies with height, time of day, season of the year, nature of the terrain, wind speeds, and temperature[10]. A number of models have been proposed for the variation of α with these variables[22]. We shall use the *linear logarithmic plot* shown in Fig. 14. This figure shows one plot for day and another plot for night, each varying with wind speed according to the equation

$$\alpha = a - b \log_{10} u(z_1) \tag{14}$$

The coefficients a and b can be determined by a *linear regression* program. Typical values of a and b are 0.11 and 0.061 in the daytime and 0.38 and 0.209 at night.

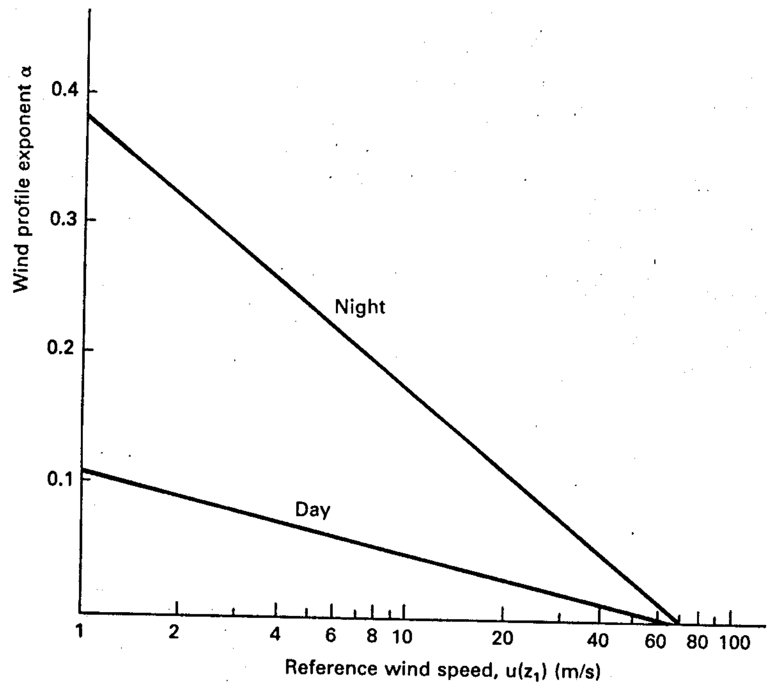

Figure 14: Variation of wind profile exponent α with reference wind speed $u(z_1)$.

Several sets of this figure can be generated at each site if necessary. One figure can be developed for each season of the year, for example. Temperature, wind direction, and height effects can also be accommodated by separate figures. Such figures would be valid at only the site where they were measured. They could be used at other sites with similar terrain with some caution, of course.

The average value of α has been determined by many measurements around the world to be about one-seventh. This has led to the common reference to Eq. 13 as the *one-seventh power law equation.* This average value should be used only if site specific data is not available, because of the wide range of values that α can assume.

6 WIND-SPEED STATISTICS

The speed of the wind is continuously changing, making it desirable to describe the wind by statistical methods. We shall pause here to examine a few of the basic concepts of probability and statistics, leaving a more detailed treatment to the many books written on the subject.

One statistical quantity which we have mentioned earlier is the average or arithmetic mean. If we have a set of numbers u_i, such as a set of measured wind speeds, the mean of the set is defined as

$$\bar{u} = \frac{1}{n}\sum_{i=1}^{n} u_i \tag{15}$$

The sample size or the number of measured values is n.

Another quantity seen occasionally in the literature is the *median.* If n is odd, the median is the middle number after all the numbers have been arranged in order of size. As many numbers lie below the median as above it. If n is even the median is halfway between the two middle numbers when we rank the numbers.

In addition to the mean, we are interested in the variability of the set of numbers. We want to find the discrepancy or deviation of each number from the mean and then find some sort of average of these deviations. The mean of the deviations $u_i - u$ is zero, which does not tell us much. We therefore square each deviation to get all positive quantities. The *variance* σ^2 of the data is then defined as

$$\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}(u_i - \bar{u})^2 \tag{16}$$

The term n - 1 is used rather than n for theoretical reasons we shall not discuss here[2].

The *standard deviation* σ is then defined as the square root of the variance.

$$\text{standard deviation} = \sqrt{\text{variance}} \tag{17}$$

Example

Five measured wind speeds are 2,4,7,8, and 9 m/s. Find the mean, the variance, and the standard deviation.

$$\bar{u} = \frac{1}{5}(2+4+7+8+9) = 6.00 \ \text{m/s}$$

$$\begin{aligned}\sigma^2 &= \frac{1}{4}[(2-6)^2+(4-6)^2+(7-6)^2+(8-6)^2+(9-6)^2] \\ &= \frac{1}{4}(34) = 8.5 \ \ \mathrm{m^2/s^2} \\ \sigma &= \sqrt{8.5} = 2.92 \ \ \mathrm{m/s}\end{aligned}$$

Wind speeds are normally measured in integer values, so that each integer value is observed many times during a year of observations. The numbers of observations of a specific wind speed u_i will be defined as m_i. The mean is then

$$\bar{u} = \frac{1}{n}\sum_{i=1}^{w} m_i u_i \tag{18}$$

where w is the number of different values of wind speed observed and n is still the total number of observations.

It can be shown[2] that the variance is given by

$$\sigma^2 = \frac{1}{n-1}\left[\sum_{i=1}^{w} m_i u_i^2 - \frac{1}{n}\left(\sum_{i=1}^{w} m_i u_i\right)^2\right] \tag{19}$$

The two terms inside the brackets are nearly equal to each other so full precision needs to be maintained during the computation. This is not difficult with most of the hand calculators that are available.

Example

A wind data acquisition system located in the tradewinds on the northeast coast of Puerto Rico measures 6 m/s 19 times, 7 m/s 54 times, and 8 m/s 42 times during a given period. Find the mean, variance, and standard deviation.

$$\begin{aligned}\bar{u} &= \frac{1}{115}[19(6)+54(7)+42(8)] = 7.20 \ \ \mathrm{m/s} \\ \sigma^2 &= \{\frac{1}{114}[19(6)^2+54(7)^2+42(8)^2 - \frac{1}{115}[19(6)+54(7)+42(8)]^2\} \\ &= \frac{1}{114}(6018-5961.600) = 0.495 \ \ \mathrm{m^2/s^2} \\ \sigma &= 0.703 \ \ \mathrm{m/s}\end{aligned}$$

Many hand calculators have a built-in routine for computing mean and standard deviation. The answers of the previous example can be checked on such a machine by anyone willing to punch in 115 numbers. In other words, Eq. 19 provides a convenient shortcut to finding the variance as compared with the method indicated by Eq. 16, or even by direct computation on a hand calculator.

Both the mean and the standard deviation will vary from one period to another or from one location to another. It may be of interest to some people to arrange these values in *rank order* from smallest to largest. We can then immediately pick out the smallest, the median, and the largest value. The terms *smallest* and *largest* are not used much in statistics because of the possibility that one value may be widely separated from the rest. The fact that the highest recorded surface wind speed is 105 m/s at Mt. Washington is not very helpful in estimating peak speeds at other sites, because of the large gap between this speed and the peak speed at the next site in the rank order. The usual practice is to talk about *percentiles*, where the 90 percentile mean wind would refer to that mean wind speed which is exceeded by only 10 % of the measured means. Likewise, if we had 100 recording stations, the 90 percentile standard deviation would be the standard deviation of station number 90 when numbered in ascending rank order from the station with the smallest standard deviation. Only 10 stations would have a larger standard deviation (or more variable winds) than the 90 percentile value. This practice of using percentiles allows us to consider cases away from the median without being too concerned about an occasional extreme value.

We shall now define the *probability* p of the discrete wind speed u_i being observed as

$$p(u_i) = \frac{m_i}{n} \tag{20}$$

By this definition, the probability of an 8 m/s wind speed being observed in the previous example would be 42/115 = 0.365. With this definition, the sum of all probabilities will be unity.

$$\sum_{i=1}^{w} p(u_i) = 1 \tag{21}$$

Note that we are using the same symbol p for both pressure and probability. Hopefully, the context will be clear enough that this will not be too confusing.

We shall also define a cumulative distribution function $F(u_i)$ as the probability that a measured wind speed will be less than or equal to u_i.

$$F(u_i) = \sum_{j=1}^{i} p(u_j) \tag{22}$$

The cumulative distribution function has the properties

$$F(-\infty) = 0, \qquad F(\infty) = 1 \tag{23}$$

Example

A set of measured wind speeds is given in Table 2.2. Find $p(u_i)$ and $F(u_i)$ for each speed. The total number of observations is $n = 211$.

Table 2.2. Wind speed histogram

i	u_i	m_i	$p(u_i)$	$F(u_i)$
1	0	0	0	0
2	1	0	0	0
3	2	15	0.071	0.071
4	3	42	0.199	0.270
5	4	76	0.360	0.630
6	5	51	0.242	0.872
7	6	27	0.128	1.000

The values of $p(u_i)$ and $F(u_i)$ are computed from Eqs. 20 and 22 and tabulated in the table.

We also occasionally need a probability that the wind speed is between certain values or above a certain value. We shall name this probability $P(u_a \leq u \leq u_b)$ where u_b may be a very large number. It is defined as

$$P(u_a \leq u \leq u_b) = \sum_{i=a}^{b} p(u_i) \tag{24}$$

For example, the probability $P(5 \leq u \leq \infty)$ that the wind speed is 5 m/s or greater in the previous example is $0.242 + 0.128 = 0.370$.

It is convenient for a number of theoretical reasons to model the wind speed frequency curve by a continuous mathematical function rather than a table of discrete values. When we do this, the probability values $p(u_i)$ become a density function $f(u)$. The density function $f(u)$ represents the probability that the wind speed is in a 1 m/s interval centered on u. The discrete probabilities $p(u_i)$ have the same meaning if they were computed from data collected at 1 m/s intervals. The area under the density function is unity, which is shown by the integral equivalent of Eq. 21.

$$\int_0^\infty f(u)du = 1 \tag{25}$$

The *cumulative distribution function* $F(u)$ is given by

$$F(u) = \int_0^u f(x)dx \tag{26}$$

The variable x inside the integral is just a dummy variable representing wind speed for the purpose of integration.

Both of the above integrations start at zero because the wind speed cannot be negative. When the wind speed is considered as a continuous random variable, the cumulative distribution function has the properties $F(0) = 0$ and $F(\infty) = 1$. The quantity $F(0)$ will not necessarily be zero in the discrete case because there will normally be some zero wind speeds measured which are included in $F(0)$. In the continuous case, however, $F(0)$ is the integral of Eq. 26 with integration limits both starting and ending at zero. Since $f(u)$ is a well behaved function at $u = 0$, the integration has to yield a result of zero. This is a minor technical point which should not cause any difficulties later.

We will sometimes need the inverse of Eq. 26 for computational purposes. This is given by

$$f(u) = \frac{dF(u)}{du} \tag{27}$$

The general relationship between $f(u)$ and $F(u)$ is sketched in Fig. 15. $F(u)$ starts at 0, changes most rapidly at the peak of $f(u)$, and approaches 1 asymptotically.

The mean value of the density function $f(u)$ is given by

$$\bar{u} = \int_0^\infty u f(u) du \tag{28}$$

The variance is given by

$$\sigma^2 = \int_0^\infty (u - \bar{u})^2 f(u) du \tag{29}$$

These equations are used to compute theoretical values of mean and variance for a wide variety of statistical functions that are used in various applications.

7 WEIBULL STATISTICS

There are several density functions which can be used to describe the wind speed frequency curve. The two most common are the *Weibull* and the *Rayleigh* functions. For the statistically inclined reader, the Weibull is a special case of the Pearson Type III or generalized gamma distribution, while the Rayleigh [or *chi with two degrees of freedom*(chi-2)] distribution is a subset of the Weibull. The Weibull is a *two parameter* distribution while the Rayleigh has only *one parameter*. This makes the Weibull somewhat more versatile and the Rayleigh somewhat simpler to use. We shall present the Weibull distribution first.

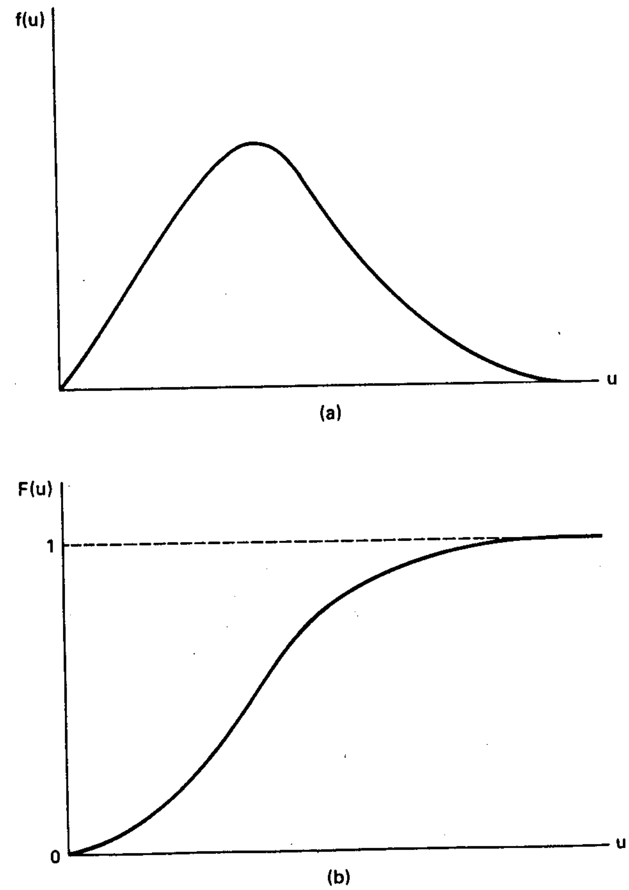

Figure 15: General relationship between (a) a density function $f(u)$ and (b) a distribution function $F(u)$

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The wind speed u is distributed as the Weibull distribution if its probability density function is

$$f(u) = \frac{k}{c}\left(\frac{u}{c}\right)^{k-1} \exp\left[-\left(\frac{u}{c}\right)^{k}\right] \qquad (k > 0, u > 0, c > 1) \tag{30}$$

We are using the expression $\exp(x)$ to represent e^x.

This is a two parameter distribution where c and k are the *scale parameter* and the *shape parameter*, respectively. Curves of $f(u)$ are given in Fig. 16, for the scale parameter $c = 1$. It can be seen that the Weibull density function gets relatively more narrow and more peaked as k gets larger. The peak also moves in the direction of higher wind speeds as k increases. A comparison of Figs. 16 and 7 shows that the Weibull has generally the right shape to fit wind speed frequency curves, at least for these two locations. Actually, data collected at many locations around the world can be reasonably well described by the Weibull density function if the time period is not too short. Periods of an hour or two or even a day or two may have wind data which are not well fitted by a Weibull or any other statistical function, but for periods of several weeks to a year or more, the Weibull usually fits the observed data reasonably well.

It may have been noticed in Fig. 16 that the wind speed only varies between 0 and 2.4 m/s, a range with little interest from a wind power viewpoint. This is not really a problem because the scale parameter c can scale the curves to fit different wind speed regimes.

If c is different from unity, the values of the vertical axis have to divided by c, as seen by Eq. 30. Since one of the properties of a probability density function is that the area under the curve has to be unity, then as the curve is compressed vertically, it has to expand horizontally. For $c = 10$, the peak value of the $k = 2.0$ curve is only 0.0858 but this occurs at a speed u of 7 rather than 0.7. If the new curve were graphed with vertical increments of 1/10 those of Fig. 16 and horizontal increments of 10 times, the new curve would have the same appearance as the one in Fig. 16. Therefore this figure may be used for any value of c with the appropriate scaling.

For k greater than unity, $f(u)$ becomes zero at zero wind speed. The Weibull density function thus cannot fit a wind speed frequency curve at zero speed because the frequency of calms is always greater than zero. This is not a serious problem because a wind turbine's output would be zero below some cut-in speed anyway. What is needed is a curve which will fit the observed data above some minimum speed. The Weibull density function is a suitable curve for this task.

A possible problem in fitting data is that the Weibull density function is defined for all values of u for $0 \leq u \leq \infty$ whereas the actual number of observations will be zero above some maximum wind speed. Fitting a nonzero function to zero data can be difficult. Again, this is not normally a serious problem because $f(u)$ goes to zero for all practical purposes for u/c greater than 2 or 3, depending on the value of k. Both ends of the curve have to receive special attention because of these possible problems, as will be seen later.

The mean wind speed $\bar{u}$ computed from Eq. 28 is

$$\bar{u} = \int_0^\infty \frac{uk}{c}\left(\frac{u}{c}\right)^{k-1} \exp\left[-\left(\frac{u}{c}\right)^k\right] du \tag{31}$$

If we make the change of variable

$$x = \left(\frac{u}{c}\right)^k \tag{32}$$

then the mean wind speed can be written

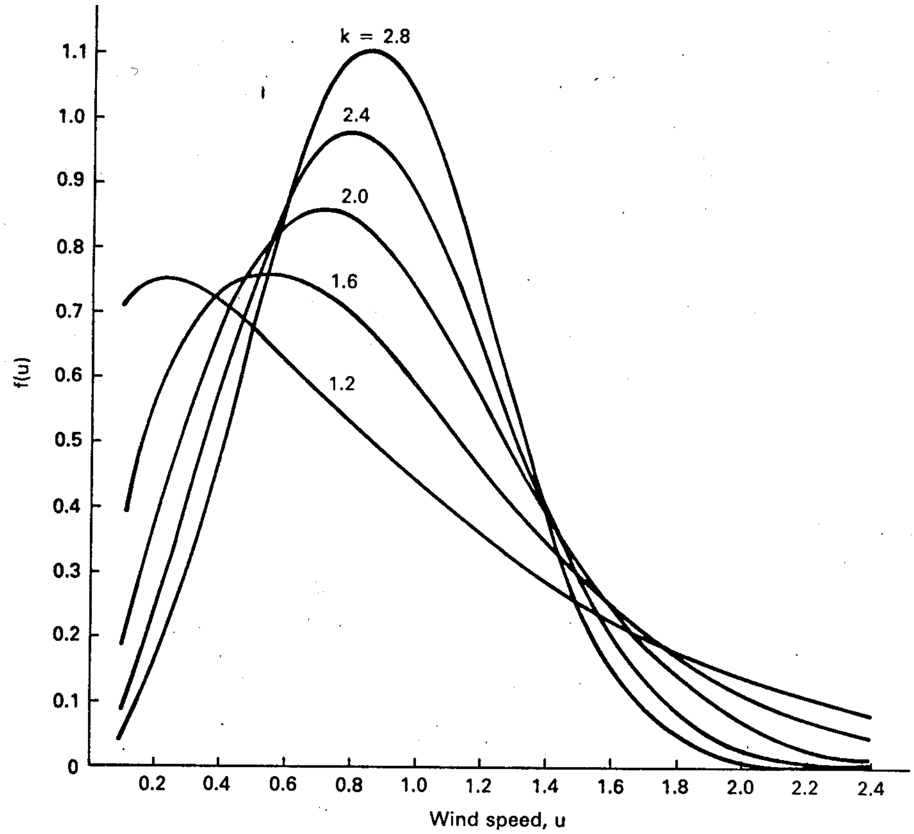

Figure 16: Weibull density function $f(u)$ for scale parameter $c = 1$.

$$\bar{u} = c\int_0^\infty x^{1/k}e^{-x}dx \tag{33}$$

This is a complicated expression which may not look very familiar to us. However, it is basically in the form of another mathematical function, the *gamma function.* Tables of values exist for the gamma function and it is also implemented in the large mathematical software packages just like trigonometric and exponential functions. The gamma function , $\Gamma(y)$, is usually written in the form

$$\Gamma(y) = \int_0^\infty e^{-x}x^{y-1}dx \tag{34}$$

Equations 33 and 34 have the same integrand if $y = 1 + 1/k$. Therefore the mean wind speed is

$$\bar{u} = c\Gamma\left(1 + \frac{1}{k}\right) \tag{35}$$

Published tables that are available for the gamma function $\Gamma(y)$ are only given for $1 \leq y \leq 2$. If an argument y lies outside this range, the recursive relation

$$\Gamma(y+1) = y\Gamma(y) \tag{36}$$

must be used. If y is an integer,

$$\Gamma(y+1) = y! = y(y-1)(y-2)\cdots(1) \tag{37}$$

The factorial $y!$ is implemented on the more powerful hand calculators. The argument y is not restricted to an integer, so the quantity computed is actually $\Gamma(y+1)$. This may be the most convenient way of calculating the gamma function in many situations.

Normally, the wind data collected at a site will be used to directly calculate the mean speed $\bar{u}$. We then want to find c and k from the data. A good estimate for c can be obtained quickly from Eq. 35 by considering the function $c/\bar{u}$ as a function of k which is given in Fig. 17. For values of k below unity, the ratio $c/\bar{u}$ decreases rapidly. For k above 1.5 and less than 3 or 4, however, the ratio $c/\bar{u}$ is essentially a constant, with a value of about 1.12. This means that the scale parameter is directly proportional to the mean wind speed for this range of k.

$$c = 1.12\bar{u} \qquad (1.5 \leq k \leq 3.0) \tag{38}$$

Most good wind regimes will have the shape parameter k in this range, so this estimate of c in terms of u will have wide application.

It can be shown by substitution that the Weibull distribution function $F(u)$ which satisfies Eq. 27, and also meets the other requirements of a distribution function, i.e. $F(0) = 0$ and $F(\infty) = 1$, is

$$F(u) = 1 - \exp\left[-\left(\frac{u}{c}\right)^k\right] \tag{39}$$

The variance of the Weibull density function can be shown to be

$$\sigma^2 = c^2\left[\Gamma\left(1 + \frac{2}{k}\right) - \Gamma^2\left(1 + \frac{1}{k}\right)\right] = (\bar{u})^2\left[\frac{\Gamma(1+2/k)}{\Gamma^2(1+1/k)} - 1\right] \tag{40}$$

The probability of the wind speed u being equal to or greater than u_a is

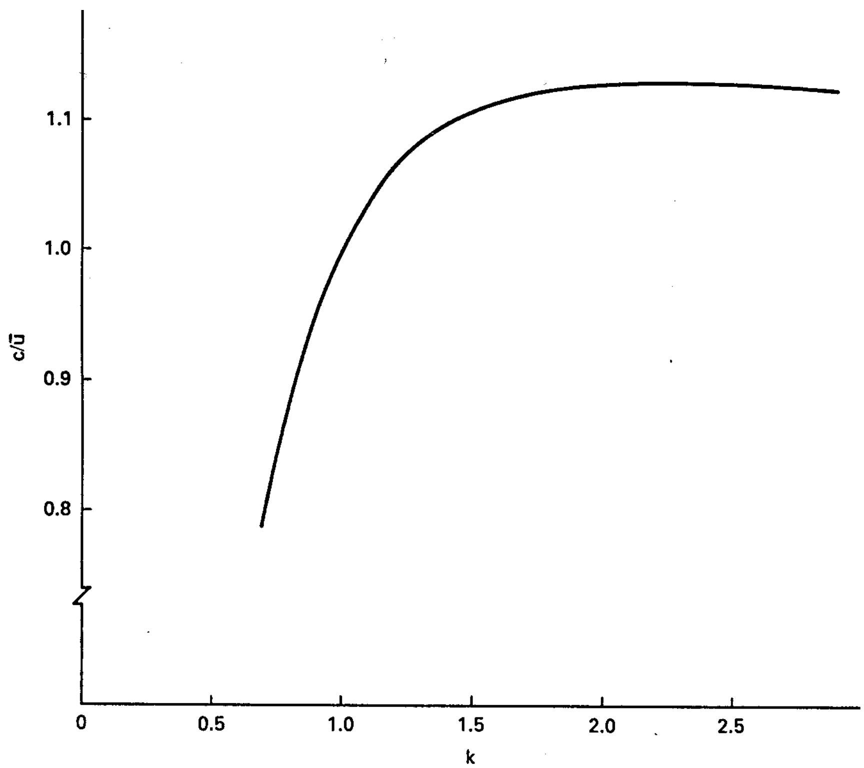

Figure 17: Weibull scale parameter c divided by mean wind speed u versus Weibull shape parameter k

$$P(u \geq u_a) = \int_{u_a}^{\infty} f(u)du = \exp\left[-\left(\frac{u_a}{c}\right)^k\right] \tag{41}$$

The probability of the wind speed being within a 1 m/s interval centered on the wind speed u_a is

$$\begin{aligned} P(u_a - 0.5 \leq u \leq u_a + 0.5) &= \int_{u_a-0.5}^{u_a+0.5} f(u)du \\ &= \exp\left[-\left(\frac{u_a - 0.5}{c}\right)^k\right] - \exp\left[-\left(\frac{u_a + 0.5}{c}\right)^k\right] \end{aligned}$$

$$\simeq \quad f(u_a)\Delta u = f(u_a) \tag{42}$$

Example

The Weibull parameters at a given site are $c = 6$ m/s and $k = 1.8$. Estimate the number of hours per year that the wind speed will be between 6.5 and 7.5 m/s. Estimate the number of hours per year that the wind speed is greater than or equal to 15 m/s.

From Eq. 42, the probability that the wind is between 6.5 and 7.5 m/s is just $f(7)$, which can be evaluated from Eq. 30 as

$$f(7) = \frac{1.8}{6}\left(\frac{7}{6}\right)^{1.8-1} \exp\left[-\left(\frac{7}{6}\right)^{1.8}\right] = 0.0907$$

This means that the wind speed will be in this interval 9.07 % of the time, so the number of hours per year with wind speeds in this interval would be

$$(0.0907)(8760) = 794 \;\; h/year$$

From Eq. 41, the probability that the wind speed is greater than or equal to 15 m/s is

$$P(u \geq 15) = \exp\left[-\left(\frac{15}{6}\right)^{1.8}\right] = 0.0055$$

which represents

$$(0.0055)(8760) = 48 \;\; \text{h/year}$$

If a particular wind turbine had to be shut down in wind speeds above 15 m/s, about 2 days per year of operating time would be lost.

We shall see in Chapter 4 that the power in the wind passing through an area A perpendicular to the wind is given by

$$P_w = \frac{1}{2}\rho A u^3 \qquad \text{W} \tag{43}$$

The average power in the wind is then

$$\bar{P}_w = \frac{1}{2}\rho A \sum_{i=1}^{w} p(u_i)u_i^3 \qquad \text{W} \tag{44}$$

We may think of this value as the true or actual power in the wind if the probabilities $p(u_i)$ are determined from the actual wind speed data.

If we model the actual wind data by a probability density function $f(u)$, then the average power in the wind is

$$\bar{P}_w = \frac{1}{2}\rho A \int_0^\infty u^3 f(u) du \qquad \text{W} \tag{45}$$

It can be shown[13] that when $f(u)$ is the Weibull density function, the average power is

$$\bar{P}_w = \frac{\rho A \bar{u}^3 \Gamma(1 + 3/k)}{2[\Gamma(1 + 1/k)]^3} \qquad \text{W} \tag{46}$$

If the Weibull density function fits the actual wind data exactly, then the power in the wind predicted by Eq. 46 will be the same as that predicted by Eq. 44. The greater the difference between the values obtained from these two equations, the poorer is the fit of the Weibull density function to the actual data.

Actually, wind speeds outside some range are of little use to a practical wind turbine. There is inadequate power to spin the turbine below perhaps 5 or 6 m/s and the turbine may reach its rated power at 12 m/s. Excess wind power is spilled or wasted above this speed so the turbine output power can be maintained at a constant value. Therefore, the quality of fit between the actual data and the Weibull model is more important within this range than over all wind speeds. We shall consider some numerical examples of these fits later in the chapter.

The function $u^3 f(u)$ starts at zero at $u = 0$, reaches a peak value at some wind speed u_{me}, and finally returns to zero at large values of u. The yearly energy production at wind speed u_i is the power in the wind times the fraction of time that power is observed times the number of hours in the year. The wind speed u_{me} is the speed which produces more energy (the product of power and time) than any other wind speed. Therefore, the maximum energy obtained from any one wind speed is

$$W_{\max} = \frac{1}{2}\rho A u_{me}^3 f(u_{me})(8760) \tag{47}$$

The turbine should be designed so this wind speed with maximum energy content is included in its best operating wind speed range. Some applications will even require the turbine to be designed with a rated wind speed equal to this maximum energy wind speed. We therefore want to find the wind speed u_{me}. This can be found by multiplying Eq. eq:2.30 by u^3, setting the derivative equal to zero, and solving for u. After a moderate amount of algebra the result can be shown to be

$$u_{me} = c\left(\frac{k+2}{k}\right)^{1/k} \qquad \text{m/s} \tag{48}$$

We see that u_{me} is greater than c so it will therefore be greater than the mean speed $\bar{u}$. If the mean speed is 6 m/s, then u_{me} will typically be about 8 or 9 m/s.

We see that a number of interesting results can be obtained by modeling the wind speed histogram by a Weibull density function. Other results applicable to the power output of wind turbines are developed in Chapter 4.

8 DETERMINING THE WEIBULL PARAMETERS

There are several methods available for determining the Weibull parameters c and k[13]. If the mean and variance of the wind speed are known, then Eqs. 35 and 40 can be solved for c and k directly. At first glance, this would seem impossible because k is buried in the argument of a gamma function. However, Justus[13] has determined that an acceptable approximation for k from Eq. 40 is

$$k = \left(\frac{\sigma}{\bar{u}}\right)^{-1.086} \qquad (49)$$

This is a reasonably good approximation over the range $1 \leq k \leq 10$. Once k has been determined, Eq. 35 can be solved for c.

$$c = \frac{\bar{u}}{\Gamma(1 + 1/k)} \qquad (50)$$

The variance of a histogram of wind speeds is not difficult to find from Eq. 19, so this method yields the parameters c and k rather easily. The method can even be used when the variance is not known, by simply estimating k. Justus[13] examined the wind speed distributions at 140 sites across the continental United States measured at heights near 10 m, and found that k appears to be proportional to the square root of the mean wind speed.

$$k = d_1\sqrt{\bar{u}} \qquad (51)$$

The proportionality constant d_1 is a site specific constant with an average value of 0.94 when the mean wind speed $\bar{u}$ is given in meters per second. The constant d_1 is between 0.73 and 1.05 for 80 % of the sites. The average value of d_1 is normally adequate for wind power calculations, but if more accuracy is desired, several months of wind data can be collected and analyzed in more detail to compute c and k. These values of k can be plotted versus $\sqrt{\bar{u}}$ on log-log paper, a line drawn through the points, and d_1 determined from the slope of the line.

Another method of determining c and k which lends itself to computer analysis, is the least squares approximation to a straight line. That is, we perform the necessary mathematical operations on Eq. 30 to linearize it and then determine c and k to minimize the least squared error between the linearized ideal curve and the actual data points of $p(u_i)$. The process is somewhat of an art and there may be more than one procedure which will yield a satisfactory result. Whether the result is satisfactory or not has to be judged by the agreement between the Weibull curve and the raw data, particularly as it is used in wind power computations.

The first step of linearization is to integrate Eq. 27. This yields the distribution function $F(u)$ which is given by Eq. 39. As can be seen in Fig. 15, $F(u)$ is more nearly describable by a straight line than $f(u)$, but is still quite nonlinear. We note that $F(u)$ contains an exponential and that, in general, exponentials are linearized by taking the logarithm. In this case, because the exponent is itself raised to a power, we must take logarithms twice.

$$\ln[-\ln(1 - F(u))] = k \ln u - k \ln c \tag{52}$$

This is in the form of an equation of a straight line

$$y = ax + b \tag{53}$$

where x and y are variables, a is the slope, and b is the intercept of the line on the y axis. In particular,

$$\begin{aligned} y &= \ln[-\ln(1 - F(u)] \\ a &= k \\ x &= \ln u \\ b &= -k \ln c \end{aligned} \tag{54}$$

Data will be expressed in the form of pairs of values of u_i and $F(u_i)$. For each wind speed u_i there is a corresponding value of the cumulative distribution function $F(u_i)$. When given values for $u = u_i$ and $F(u) = F(u_i)$ we can find values for $x = x_i$ and $y = y_i$ in Eqs. 55. Being actual data, these pairs of values do not fall exactly on a straight line, of course. The idea is to determine the values of a and b in Eq. 53 such that a straight line drawn through these points has the best possible fit. It can be shown that the proper values for a and b are

$$a = \frac{\displaystyle\sum_{i=1}^{w} x_i y_i - \frac{\sum_{i=1}^{w} x_i \sum_{i=1}^{w} y_i}{w}}{\displaystyle\sum_{i=1}^{w} x_i^2 - \frac{\left(\sum_{i=1}^{w} x_i\right)^2}{w}} = \frac{\displaystyle\sum_{i=1}^{w}(x_i - \bar{x})(y_i - \bar{y})}{\displaystyle\sum_{i=1}^{w}(x_i - \bar{x})^2} \tag{55}$$

$$b = \bar{y}_i - a\bar{x}_i = \frac{1}{w}\sum_{i=1}^{w} y_i - \frac{a}{w}\sum_{i=1}^{w} x_i \tag{56}$$

In these equations $\bar{x}$ and $\bar{y}$ are the mean values of x_i and y_i, and w is the total number of pairs of values available. The final results for the Weibull parameters are

$$\begin{aligned} k &= a \\ c &= \exp\left(-\frac{b}{k}\right) \end{aligned} \tag{57}$$

One of the implied assumptions of the above process is that each pair of data points is equally likely to occur and therefore would have the same weight in determining the equation of the line. For typical wind data, this means that one reading per year at 20 m/s has the same weight as 100 readings per year at 5 m/s. To remedy this situation and assure that we have the best possible fit through the range of most common wind speeds, it is possible to redefine a weighted coefficient a in place of Eq. 55 as

$$a = \frac{\sum_{i=1}^{w} p^2(u_i)(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{w} p^2(u_i)(x_i - \bar{x})^2} \tag{58}$$

This equation effectively multiplies each x_i and each y_i by the probability of that x_i and that y_i occurring. It usually gives a better fit than the unweighted a of Eq. 55.

Eqs. 55, 56, and 58 can be evaluated conveniently on a programmable hand held calculator. Some of the more expensive versions contain a built-in linear regression function so Eqs. 55 and 56 are handled internally. All that needs to be entered are the pairs of data points. This linear regression function can be combined with the programming capability to evaluate Eq. 58 more conveniently than by separately entering each of the repeating data points m_i times.

Example

The actual wind data for Kansas City and Dodge City for the year 1970 are given in Table 2.3. The wind speed u_i is given in knots. Calm includes 0 and 1 knot because 2 knots are required to spin the anemometer enough to give a non zero reading. The parameter x_i is $\ln(u_i)$ as given in Eq. 55. The number m_i is the number of readings taken during that year at each wind speed. The total number of readings n at each site was 2912 because readings were taken every three hours. The function $p(u_i)$ is the measured probability of each wind speed at each site as given by Eq. 20. Compute the Weibull parameters c and k using the linearization method.

Table 2.3. 1970 Wind Data

		Kansas City				Dodge City			
u_i	x_i	m_i	$p(u_i)$	$F(u)$	y_i	m_i	$p(u_i)$	$F(u)$	y_i
1	0	300	0.103	0.103	-2.22	47	0.016	0.016	-4.12
2	0.69	161	0.055	0.158	-1.765	0.002	0.018	-4.02	
3	1.10	127	0.044	0.202	-1.49	82	0.028	0.046	-3.05
4	1.39	261	0.090	0.292	-1.06	65	0.022	0.068	-2.65
5	1.61	188	0.065	0.356	-0.82	140	0.048	0.116	-2.09
6	1.79	294	0.101	0.457	-0.49	219	0.075	0.92	-1.55
7	1.95	151	0.052	0.509	-0.34	266	0.091	0.283	-1.10
8	2.08	347	0.119	0.628	-0.01	276	0.095	0.378	-0.75
9	2.20	125	0.043	0.671	0.11	198	0.068	0.446	-0.53
10	2.30	376	0.129	0.800	0.48	314	0.108	0.554	-0.22
11	2.40	67	0.023	0.823	0.55	155	0.053	0.607	-0.07
12	2.48	207	0.071	0.894	0.81	177	0.061	0.668	0.10
13	2.56	67	0.023	0.917	0.91	141	0.048	0.716	0.23
14	2.64	91	0.031	0.948	1.09	142	0.049	0.765	0.37
15	2.71	29	0.010	0.958	1.16	133	0.046	0.810	0.51
16	2.77	51	0.017	0.976	1.32	96	0.033	0.843	0.62
17	2.83	19	0.006	0.982	1.40	102	0.035	0.878	0.75
18	2.89	39	0.013	0.996	1.70	101	0.035	0.913	0.89
19	2.94	1	0	0.996	1.72	48	0.016	0.930	0.98
20	3.00	7	0.002	0.999	1.89	78	0.027	0.956	1.14
21	3.05	0	0	0.999	1.89	28	0.010	0.966	1.22
22	3.09	2	0.001	0.999	1.97	21	0.007	0.973	1.29
23	3.13	0	0	0.999	1.97	23	0.008	0.981	1.38
24	3.18	1	0	1.000	2.08	12	0.004	0.985	1.44
25	3.22	1	0	1.000	-	19	0.006	0.992	1.57
26	3.26	0	0			8	0.003	0.995	1.65
27	3.30	0	0			2	0.001	0.995	1.67
28	3.33	0	0			9	0.003	0.998	1.85
>28		0	0			5			
		2912				2912			

We first use Eq. 22 to compute $F(u_i)$ for each u_i and Eqs 55 to compute y_i for each $F(u_i)$. These are listed in Table 2.3 as well as the original data.

We are now ready to use Eqs. 58 and 56 to find a and b. First, however, we plot the point pairs (x_i, y_i) for each u_i as shown in Fig. 18 for both sites. The readings for calm (0 and 1 knot) are assumed to be at 1 knot so that $x_i = \ln u_i$ is zero rather than negative infinity.

Placing a straight edge along the sets of points shows the points to be in reasonable alignment except for calm and 2 knots for Kansas City, and calm for Dodge City. As mentioned earlier, the goal is to describe the data mathematically over the most common wind speeds. The Weibull function is zero for wind speed u equal to zero (if $k > 1$) so the Weibull cannot describe calms. Therefore, it is desirable to ignore calms and perhaps 2 knots in order to get the best fit over the wind speeds of greater interest.

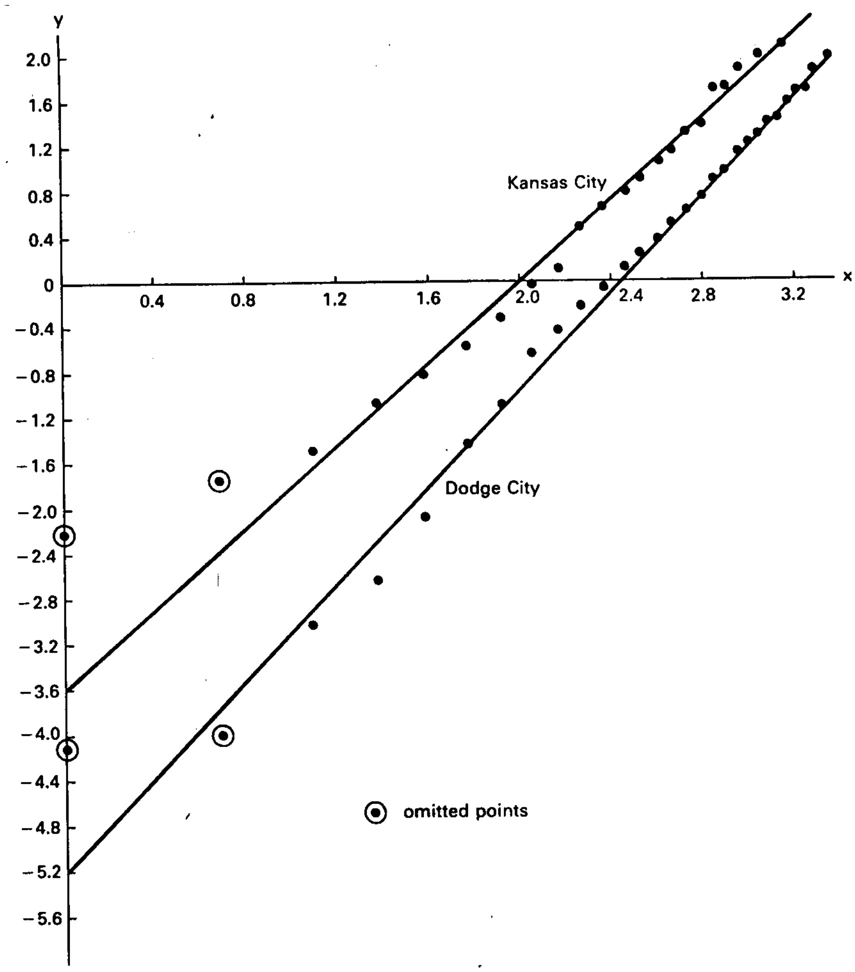

Figure 18: y versus x for Kansas City and Dodge City, 1970

A word of caution is appropriate about ignoring data points. Note that we do not want to change the location of any of the points on Fig. 18. We therefore compute $F(u_i), x_i$, and y_i from the original data set and do not adjust or renormalize any of these values. The summations of Eqs. 55, 56, and 58 are corrected by running the summation from $i = 3$ to w rather than $i = 1$ to w, if $u_i = 1$ and $u_i = 2$ are ignored. The corrected values for $\bar{x}$ and $\bar{y}$ as computed from the summations in Eq. 56 are then used in Eqs. 55 or 58. If there are no readings at a particular u_i, then the summations should just skip this value of i.

The data for Kansas City were processed for $i = 3$ to 20 and for Dodge City for $i = 3$ to 28. These upper limits include about 99.8 % of the data points and should therefore give adequate results. The expression for y in Eqs. 55 becomes undefined for $F(u)$ equal to unity so the last data point cannot be included unless $F(u)$ for this last point is arbitrarily set to something less than unity, say 0.998. The results for Kansas City are $c = 7.65$ knots and $k = 1.776$, and $c = 11.96$ knots and $k = 2.110$ for Dodge City. The corresponding best fit lines are shown in Fig. 18. It is evident that these lines fit the plotted points rather well.

There may be some who are curious about the fit of the Weibull density function to the original wind speed histogram with the c and k computed in this example. The values for $p(u_i)$ for Dodge City are shown in Fig. 19 as well as the curve of $f(u)$ computed from Eq. 30. It can be seen that the data points are rather scattered but the Weibull density function does a reasonable job of fitting the scattered points.

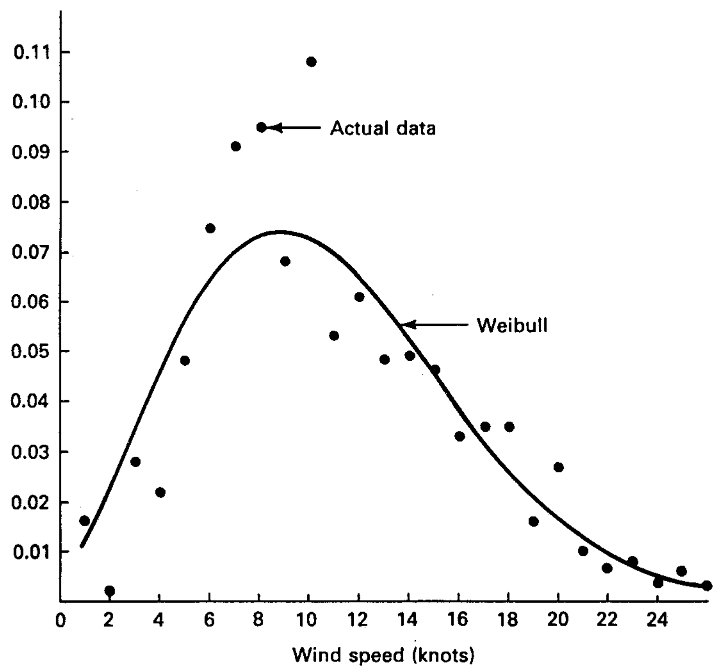

Figure 19: Actual wind data and weighted Weibull density function for Dodge City, 1970. (Data from National Weather Service.)

The data in Fig. 19 were recorded by human observation of a wind speed indicator which is continuously changing[25]. There is a human tendency to favor even integers and multiples of five when reading such an indicator. For this data set, a speed of 8 knots was recorded 276 times during the year, 9 knots 198 times, 10 knots 314 times, and 11 knots 155 times. It can be safely assumed that several readings of 8 and 10 knots should have actually been 9 or 11 knots. An automatic recording system without human bias should give a smoother set of data. The system with a human observer has excellent reliability, however, and a smoother data set really makes little difference in wind power calculations. The existing system should therefore not be changed just to get smoother data.

9 RAYLEIGH AND NORMAL DISTRIBUTIONS

We now return to a discussion of the other popular probability density function in wind power studies, the *Rayleigh* or *chi-2*. The Rayleigh probability density function is given by

$$f(u) = \frac{\pi u}{2\bar{u}^2} \exp\left[-\frac{\pi}{4}\left(\frac{u}{\bar{u}}\right)^2\right] \tag{59}$$

The Rayleigh cumulative distribution function is

$$F(u) = 1 - \exp\left[-\frac{\pi}{4}\left(\frac{u}{\bar{u}}\right)^2\right] \tag{60}$$

The probability that the wind speed u is greater than or equal to u_a is just

$$P(u \geq u_a) = 1 - F(u_a) = \exp\left[-\frac{\pi}{4}\left(\frac{u}{\bar{u}}\right)^2\right] \tag{61}$$

The variance of this density function is

$$\sigma^2 = \left(\frac{4}{\pi} - 1\right)\bar{u}^2 \tag{62}$$

It may be noted that the variance is only a function of the mean wind speed. This means that one important statistical parameter is completely described in terms of a second quantity, the mean wind speed. Since the mean wind speed is always computed at any measurement site, all the statistics of the Rayleigh density function used to describe that site are immediately available without massive amounts of additional computation. As mentioned earlier, this makes the Rayleigh density function very easy to use. The only question is the quality of results. It appears from some studies[5, 6], that the Rayleigh will yield acceptable results in most cases. The natural variability in the wind from year to year tends to limit the need for the greater sophistication of the Weibull density function.

It should be emphasized that actual histograms of wind speeds may be difficult to fit by any mathematical function, especially if the period of time is short. This is illustrated by Fig. 20, which shows actual data for a site and the Weibull and Rayleigh models of the data. The wind speed was automatically measured about 245,000 times over a 29 day period in July, 1980, between 7:00 and 8:00 p.m. to form the histogram. A bimodal (two-humped) characteristic is observed, since 3 m/s and 9 m/s are observed more often than 5 m/s. This is evidently due to the wind speed being high for a number of days and relatively low the other days, with few days actually having average wind speeds.

The Weibull is seen to be higher than the Rayleigh between 5 and 12 m/s and lower outside this range. Both functions are much higher than the actual data above 12 m/s. The actual

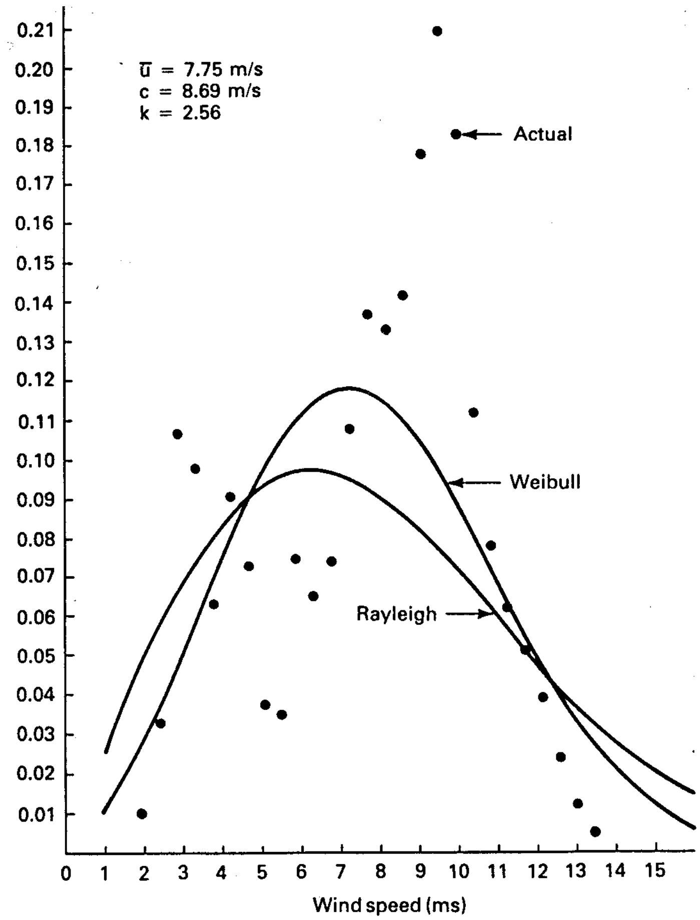

Figure 20: Actual wind data, and Weibull and Rayleigh density functions for Tuttle Creek, 7:00–8:00 p.m., July 1980, 50 m.

wind power density as computed from Eq. 44 for standard conditions is $\bar{P}_w/A = 396$ W/m^2. The power density computed from the Weibull model, Eq. 46, is 467 W/m^2, while the power density computed from the Rayleigh density function by a process similar to Eq. 44 is 565 W/m^2. The Weibull is 18 % high while the Rayleigh is 43 % high.

If we examine just the wind speed range of 5-12 m/s, we get an entirely different picture. The actual wind power density in this range is $\bar{P}_w/A = 362$ W/m^2, while the Weibull predicts a density of 308 W/m^2, 15 % low, and the Rayleigh predicts 262 W/m^2, 28 % low. This shows that neither model is perfect and that results from such models need to be used with caution. However, the Weibull prediction is within 20 % of the actual value for either wind speed range, which is not bad for a data set that is so difficult to mathematically describe.

Another example of the ability of the Weibull and Rayleigh density functions to fit actual data is shown in Fig. 21. The mean speed is 4.66 m/s as compared with 7.75 m/s in Fig. 20

and k is 1.61 as compared with 2.56. This decrease in k to a value below 2 causes the Weibull density function to be below the Rayleigh function over a central range of wind speeds, in this case 2–8 m/s. The actual data are concentrated between 2 and 4 m/s and neither function is able to follow this wide variation. The actual wind power density in this case is 168 W/m^2, while the Weibull prediction is 161 W/m^2, 4 % low, and the Rayleigh prediction is 124 W/m^2, 26 % low. Considering only the 5–12 m/s range, the actual power density is 89 W/m^2, the Weibull prediction is 133 W/m^2, 49 % high, and the Rayleigh prediction is 109 W/m^2, 22 % high.

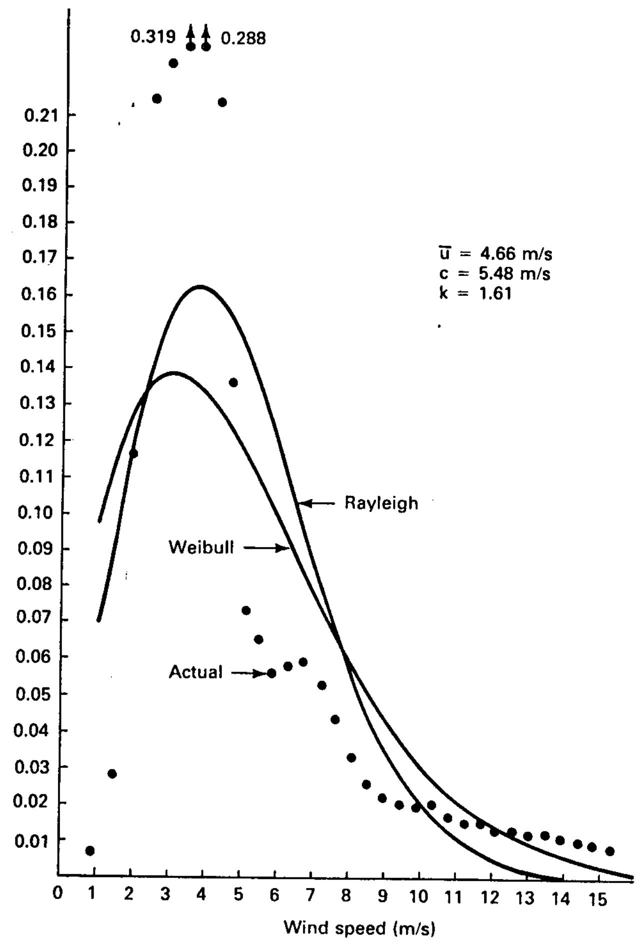

Figure 21: Actual wind data, and Weibull and Rayleigh density functions for Tuttle Creek, 7:00–8:00 a.m., August 1980, 10 m.

We see from these two figures that it is difficult to make broad generalizations about the ability of the Weibull and Rayleigh density functions to fit actual data. Either one may be either high or low in a particular range. The final test or proof of the usefulness of these functions will be in their ability to predict the power output of actual wind turbines. In the meantime it appears that either function may yield acceptable results, with the Weibull being more accurate and the Rayleigh easier to use.

Although the actual wind speed distribution can be described by either a Weibull or a Rayleigh density function, there are other quantities which are better described by a *normal distribution*. The distribution of monthly or yearly mean speeds is likely to be normally distributed around a long-term mean wind speed, for example. The normal curve is certainly the best known and most widely used distribution for a continuous random variable, so we shall mention a few of its properties.

The density function $f(u)$ of a normal distribution is

$$f(u) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(u-\bar{u})^2}{2\sigma^2}\right] \tag{63}$$

where $\bar{u}$ is the mean and σ is the standard deviation. In this expression, the variable u is allowed to vary from $-\infty$ to $+\infty$. It is physically impossible for a wind speed to be negative, of course, so we cannot forget the reality of the observed quantity and follow the mathematical model past this point. This will not usually present any difficulty in examining mean wind speeds.

The cumulative distribution function $F(u)$ is given by

$$F(u) = \int_{-\infty}^{u} f(x)dx \tag{64}$$

This integral does not have a simple closed form solution, so tables of values are determined from approximate integration methods. The variable in this table is usually defined as

$$q = \frac{u-\bar{u}}{\sigma} \qquad \text{or} \qquad u = \bar{u} + q\sigma \tag{65}$$

Thus q is the number of standard deviations that u is away from $\bar{u}$. A brief version of this table is shown in Table 2.4. We see from the table, for example, that $F(u)$ for a wind speed one standard deviation below the mean is 0.159. This means there is a 15.9 % probability that the mean speed for any period of interest will be more than one standard deviation below the long term mean. Since the normal density function is symmetrical, there is also a 15.9 % probability that the mean speed for some period will be more than one standard deviation above the long term mean.

Table 2.4. Normal CumulativeDistribution Function

q	$F(u)$	q	$F(u)$
-3.090	0.001	+0.43	0.666
-2.326	0.01	+0.675	0.75
-2.054	0.02	+0.842	0.8
-2.00	0.023	+1.0	0.841
-1.645	0.05	+1.281	0.9
-1.281	0.1	+1.645	0.95
-1.0	0.159	+2.0	0.977
-0.842	0.2	+2.054	0.98
-0.675	0.25	+2.326	0.99
-0.43	0.334	+3.090	0.999
0	0.5		

As we move further away from the mean, the probability decreases rapidly. There is a 2.3 % probability that the mean speed will be smaller than a value two standard deviations below the long term mean, and only 0.1 % probability that it will be smaller than a value three standard deviations below the long term mean. This gives us a measure of the width of a normal distribution.

It is also of interest to know the probability of a given data point being within a certain distance of the mean. A few of these probabilities are shown in Table 2.5. For example, the fraction 0.6827 of all measured values fall within one standard deviation of the mean if they are normally distributed. Also, 95 % of all measured values fall within 1.96 standard deviations of the mean.

We can define the em 90 percent confidence interval as the range within ± 1.645 standard deviations of the mean. If the mean is 10 m/s and the standard deviation happens to be 1 m/s, then the 90 % confidence interval will be between 8.355 and 11.645 m/s. That is, 90 % of individual values would be expected to lie in this interval.

Table 2.5. Probability of finding the variable within q standard deviations of $\bar{u}$.

q	$P(\|u - \bar{u}\| \leq q\sigma)$
1.0	0.6827
1.645	0.9000
1.960	0.9500
2.0	0.9545
2.576	0.9900
3.0	0.9973

Example

The monthly mean wind speeds at Dodge City for 1958 were 11.78, 13.66, 11.16, 12.94, 12.10, 13.47, 12.56, 10.86, 13.77, 11.76, 12.44, and 12.55 knots. Find the yearly mean (assuming all months have the same number of days), the standard deviation, and the wind speeds one and two standard deviations from the mean. What monthly mean will be exceeded 95 % of the time? What is the 90 % confidence interval?

By a hand held calculator, we find

$$\bar{u} = 12.42$$

$$\sigma = 0.94$$

The wind speeds one standard deviation from the mean are 11.48 and 13.36 knots, while the speeds two standard deviations from the mean are 10.54 and 14.30 knots. From Table 2.4 we see that $F(u)$ = 0.05 (indicating 95 % of the values are larger) for q = -1.645. From Eq. 65 we find

$$u = 12.42 + (-1.645)(0.94) = 10.87 \text{ knots}$$

Based on this one year's data we can say that the monthly mean wind speed at Dodge City should exceed 10.87 knots (5.59 m/s) for 95 % of all months.

The 90 % confidence interval is given by the interval 12.42 ± 1.645(0.94) or between 10.87 and 13.97 knots. We would expect from this analysis that 9 out of 10 monthly means would be in this interval. In examining the original data set, we find that only one month out of 12 is outside the interval, and it is just barely outside. This type of result is rather typical with such small data sets. If we considered a much larger data set such as a 40 year period with 480 monthly means, then we could expect approximately 48 months to actually fall outside this 90 % confidence interval.

We might now ask ourselves how confident we are in the results of this example. After all, only one year's wind data were examined. Perhaps we picked an unusual year with mean and standard deviation far removed from their long term averages. We need to somehow specify the confidence we have in such a result.

Justus, Mani and Mikhail examined long term wind data[14] for 40 locations in the United States, including Alaska, Hawaii, and Wake Island. All sites had ten or more years of data from a fixed anemometer location and a long term mean wind speed of 5 m/s or greater. They found that monthly and yearly mean speeds are distributed very closely to a normal or Gaussian distribution, as was mentioned earlier.

The monthly means were distributed around the long term measured monthly mean $\bar{u}_m$ with an average standard deviation of $0.098\bar{u}_m$ where $\bar{u}_m$ is the mean wind speed for a given month of the year, e.g. all the April average wind speeds are averaged over the entire period of observation to get a long term average for that month. For a normal distribution the 90 % confidence interval would be, using Table 2.5, $\bar{u}_m \pm 1.645(0.098)\bar{u}_m$ or the interval between $0.84\bar{u}_m$ and $1.16\bar{u}_m$. We can therefore say that we have 90 % confidence that a measured monthly mean speed will fall in the interval $0.84\bar{u}_m$ to $1.16\bar{u}_m$. If we say that each measured monthly mean lies in this interval, we will be correct 90 times out of 100, and wrong 10 times .

The above argument applies *on the average*. That is, it is valid for sites with an average standard deviation. We cannot be as confident of sites with more variable winds and hence higher standard deviations. Since we do not know the standard deviation of the wind speed at the candidate site, we have to allow for the possibility of it being a larger number. According to Justus[14], 90 % of all observed monthly standard deviations were less than $0.145\bar{u}_m$. This is the *90 percentile level*. The appropriate interval is now $\bar{u}_m \pm 1.645(0.145)\bar{u}_m$ or the interval between $0.76\bar{u}_m$ and $1.24\bar{u}_m$. That is, any single monthly mean speed at this highly variable site will fall within the interval $0.76\bar{u}_m$ and $1.24\bar{u}_m$ with 90 % confidence. The converse is also true. That is, if we designate the mean for one month as $\bar{u}_{m1}$, the unknown long term monthly mean $\bar{u}_m$ will fall within the interval $0.76\bar{u}_{m1}$ and $1.24\bar{u}_{m1}$ with 90 % confidence. We shall try to clarify this statement with Fig. 22.

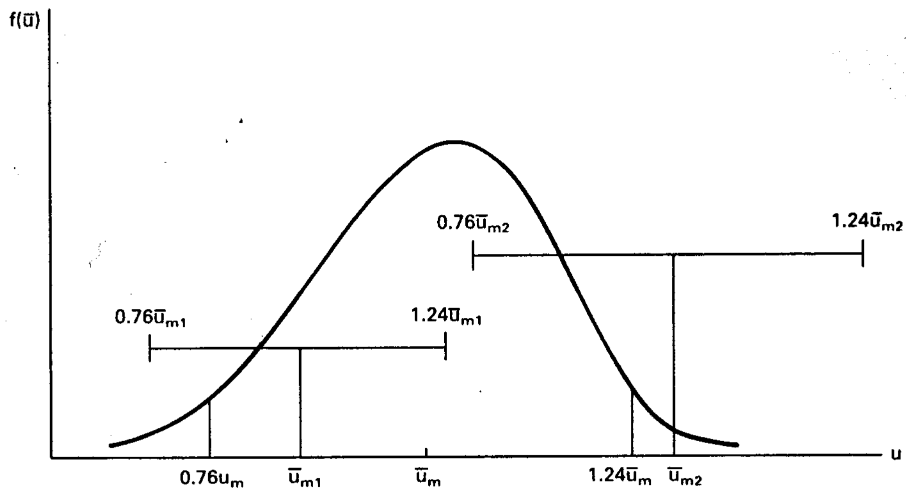

Figure 22: Confidence intervals for two measured monthly means, $\bar{u}_{m1}$ and $\bar{u}_{m2}$.

Suppose that we measure the average monthly wind speed for April during two successive years and call these values $\bar{u}_{m1}$ and $\bar{u}_{m2}$. These are shown in Fig. 22 along with the intervals $0.76\bar{u}_{m1}$ to $1.24\bar{u}_{m1}$, $0.76\bar{u}_{m2}$ to $1.24\bar{u}_{m2}$, and $0.76\bar{u}_m$ to $1.24\bar{u}_m$. The confidence interval centered on $\bar{u}_m$ contains 90 % of measured means for individual months. The mean speed $\bar{u}_{m1}$ is below $\bar{u}_m$ but its confidence interval includes $\bar{u}_m$. The mean speed $\bar{u}_{m2}$ is outside the confidence interval for $\bar{u}_m$ and the reverse is also true. If we say that the true mean $\bar{u}_m$ is between $0.76\bar{u}_{m2}$ and $1.24\bar{u}_{m2}$, we will be wrong. But since only 10 % of the individual monthly means are outside the confidence interval for $\bar{u}_m$, we will be wrong only 10 % of the time. Therefore we can say with 90 % confidence that $\bar{u}_m$ lies in the range of $0.76\bar{u}_{mi}$ to $1.24\bar{u}_{mi}$ where $\bar{u}_{mi}$ is some measured monthly mean.

Example

The monthly mean speeds for April in Dodge City for the years 1948 through 1973 are: 15.95, 12.88, 13.15, 14.10, 13.31, 15.14, 14.82, 15.58, 13.95, 14.76, 12.94, 15.03, 16.57, 13.88, 10.49, 11.30, 13.80, 10.99, 12.47, 14.40, 13.18, 12.04, 12.52, 12.05, 12.62, and 13.15 knots. Find the mean $\bar{u}_m$ and the normalized standard deviation $\sigma_m/\bar{u}_m$. Find the 90 % confidence intervals for $\bar{u}_m$, using the 50 percentile standard deviation ($\sigma_m/\bar{u}_m = 0.098$), the 90 percentile standard deviation ($\sigma_m/\bar{u}_m = 0.145$), and the actual $\sigma_m/\bar{u}_m$. How many years fall outside each of these confidence intervals? Also find the confidence intervals for the best and worst months using $\sigma_m/\bar{u}_m = 0.145$.

From a hand held calculator, the mean speed $\bar{u}_m = 13.50$ knots and the standard deviation $\sigma_m =$ 1.52 knots are calculated. The normalized standard deviation is then

$$\frac{\sigma_m}{\bar{u}_m} = 0.113$$

This is above the average, indicating that Dodge City has rather variable winds.

The 90 % confidence interval using the 50 percentile standard deviation is between $0.84\bar{u}_m$ and $1.16\bar{u}_m$, or between 11.34 and 15.66 knots. Of the actual data, 2 months have means above this interval, with 21 or 81 % of the monthly means inside the interval.

The 90 % confidence interval using the nationwide 90 percentile standard deviation is between $0.76\bar{u}_m$ and $1.24\bar{u}_m$, or between 10.26 and 16.74 knots. All the monthly means fall within this interval.

The 90 % confidence interval using the actual standard deviation of the site is between the limits $\bar{u}_m \pm 1.645(0.113)\bar{u}_m$ or between 10.99 and 16.01 knots. Of the actual data, 24 months or 92 % of the data points fall within this interval.

If only the best month was measured, the 90 % confidence interval using the 90 percentile standard deviation would be $16.57 \pm (0.24)16.57$ or between 12.59 and 20.55 knots. The corresponding interval for the worst month would be $10.49 \pm (0.24)10.49$ or between 7.97 and 13.01 knots. The latter case is the only monthly confidence interval which does not contain the long term mean of 13.5 knots. This shows that we can make confident estimates of the possible range of wind speeds from one month's data without undue concern about the possibility of that one month being an extreme value.

The 90 % confidence interval for monthly means is rather wide. Knowing that the monthly mean speed will be between 11 and 16 knots (5.7 and 8.2 m/s), as in the previous example, may not be of much help in making economic decisions. We will see in Chapter 4 that the energy output of a given turbine will increase by a factor of two as the monthly mean speed increases from 5.7 to 8.2 m/s. This means that a turbine which is a good economic choice in a 8.2 m/s wind regime may not be a good choice in a 5.7 m/s regime. Therefore, one month of wind speed measurements at a candidate site will not be enough, in many cases. The confidence interval becomes smaller as the number of measurements grows.

The wind speed at a given site will vary seasonally because of differences in large scale weather patterns and also because of the local terrain at the site. Winds from one direction may experience an increase in speed because of the shape of the hills nearby, and may experience a decrease from another direction. It is therefore advisable to make measurements of speed over a full yearly cycle. We then get a *yearly mean speed* $\bar{u}_y$. Each yearly mean speed $\bar{u}_y$ will be approximately normally distributed around the long term mean speed $\bar{u}$. Justus[14] found that the 90 % confidence interval for the yearly means was between $0.9\bar{u}$ and

$1.1\bar{u}$ for the median (50 percentile) standard deviation and between $0.85\bar{u}$ and $1.15\bar{u}$ for the 90 percentile standard deviation. As expected, these are narrower confidence intervals than were observed for the single monthly mean.

Example

The yearly mean speed at Dodge City was 11.44 knots at 7 m above the ground level in 1973. Assume a 50 percentile standard deviation and determine the 90 % confidence interval for the true long term mean speed.

The confidence interval extends from 11.44/1.1 = 10.40 to 11.44/0.9 = 12.71 knots. We can therefore say that the long term mean speed lies between 10.40 and 12.71 knots with 90 % confidence.

Our estimate of the long term mean speed by one year's data can conceivably be improved by comparing our one year mean speed to that of a nearby National Weather Service station. If the yearly mean speed at the NWS station was higher than the long term mean speed there, the measured mean speed at the candidate site can be adjusted upward by the same factor. This assumes that all the winds within a geographical region of similar topography and a diameter up to a few hundred kilometers will have similar year to year variations. If the long term mean speed at the NWS station is $\bar{u}$, the mean for one year is $\bar{u}_a$, and the mean for the same year at the candidate site is $\bar{u}_b$, then the corrected or estimated long term mean speed $\bar{u}_c$ at the candidate site is

$$\bar{u}_c = \bar{u}_b \frac{\bar{u}}{\bar{u}_a} \tag{66}$$

Note that we do not know how to assign a confidence interval to this estimate. It is a single number whose accuracy depends on both the accuracy of $\bar{u}$ and the correlation between $\bar{u}_a$ and $\bar{u}_b$. The accuracy of $\bar{u}$ should be reasonably good after 30 or more years of measurements, as is common in many NWS stations. However, the assumed correlation between $\bar{u}_a$ and $\bar{u}_b$ may not be very good. Justus[14] found that the correlation was poor enough that the estimate of long term means was not improved by using data from nearby stations. Equally good results would be obtained by applying the 90 % confidence interval approach as compared to using Eq. 66. The reason for this phenomenon was not determined. One possibility is that the type of anemometer used by the National Weather Service can easily get dirty and yield results that are low by 10 to 20 % until the next maintenance period. One NWS station may have a few months of low readings one year while another NWS station may have a few months of low readings the next year, due to the measuring equipment rather than the wind. This would make a correction like Eq. 66 very difficult to use. If this is the problem, it can be reduced by using an average of several NWS stations and, of course, by more frequent maintenance. Other studies are necessary to clarify this situation.

The actual correlation between two sites can be defined in terms of a *correlation coefficient* r, where

$$r = \frac{\sum_{i=1}^{w}(\bar{u}_{ai} - \bar{u}_a)(\bar{u}_{bi} - \bar{u}_b)}{\sqrt{\sum_{i=1}^{w}(\bar{u}_{ai} - \bar{u}_a)^2 \sum_{i=1}^{w}(\bar{u}_{bi} - \bar{u}_b)^2}} \tag{67}$$

In this expression, $\bar{u}_a$ is the long term mean speed at site a, $\bar{u}_b$ is the long term mean speed at site b, $\bar{u}_{bi}$ is the observed monthly or yearly mean at site b, and w is the number of months or years being examined. We assume that the wind speed at site b is linearly related to the speed at site a. We can then plot each mean speed $\bar{u}_{bi}$ versus the corresponding $\bar{u}_{ai}$. We then find the best straight line through this cluster of points by a least squares or linear regression process. The correlation coefficient then describes how closely our data fits this straight line. Its value can range from $r = +1$ to $r = -1$. At $r = +1$, the data falls exactly onto a straight line with positive slope, while at $r = -1$, the data falls exactly onto a straight line with negative slope. At $r = 0$, the data cannot be approximated at all by a straight line.

Example

The yearly mean wind speeds for Dodge City, Wichita, and Russell, Kansas are given in Table 2.6. These three stations form a triangle in central Kansas with sides between 150 and 220 km in length. Dodge City is west of Wichita, and Russell is northwest of Wichita and northeast of Dodge City. The triangle includes some of the best land in the world for growing hard red winter wheat. There are few trees outside of the towns and the land is flat to gently rolling in character. Elevation changes of 10 to 20 m/km are rather typical. We would expect a high correlation between the sites, based on climate and topography.

There were several anemometer height changes over the 26 year period, so in an attempt to eliminate this bias in the data, all yearly means were extrapolated to a 30 m height using the power law and an average exponent of 0.143. As mentioned earlier in this chapter, this assumption should be acceptable for these long term means.

A plot of Russell wind speeds versus Dodge City speeds is given in Fig. 23, and a similar plot for Russell versus Wichita is given in Fig. 24. A least squares fit to the data was computed using a hand held calculator with linear regression and correlation coefficient capability. The equation of the straight line through the points in Fig. 23 is $\bar{u}_b = 0.729\bar{u}_a + 3.59$ with $r = 0.596$. The corresponding equation for Fig. 24 is $\bar{u}_b = 0.181\bar{u}_a + 11.74$ with $r = 0.115$. Straight lines are drawn on these figures for these equations.

It can be seen from both the figures and the correlation coefficients that Russell winds are poorly predicted by Dodge City winds and are hardly predicted at all by Wichita winds. Years with mean speeds around 14 knots at Wichita display mean speeds between 12.3 and 16 knots at Russell. This gives further support to the conclusion made by Justus that one year's data with a 90 % confidence interval at a candidate site is just as good as extrapolating from adjacent sites.

In examining the data in Table 2.6, one notices some clusters of low wind speed years and high wind speed years. That is, it appears that a low annual mean wind may persist through the following year, and likewise for a high annual mean wind. If this is really the case, we may select a site for a wind turbine in spite of a poor wind year, and then find the performance of

Table 2.6 Average yearly speeds corrected to 30 m (100 ft)

Year	Dodge City	Wichita	Russell
1948	14.98	14.09	14.14[a]
1949	13.53	13.35	13.08[b]
1950	12.89	12.77	14.11
1951	13.83	13.47	13.07
1952	14.22	13.97	12.31
1953	15.57	14.45[c]	13.37
1954	15.20	14.98	12.83
1955	14.83	15.22	14.89
1956	14.60	14.86	14.85
1957	14.71	13.60	15.38
1958	13.85	13.74	13.63
1959	14.91	15.21	14.88
1960	15.23	14.55	13.67
1961	12.37	13.29	12.45
1962	13.53	13.18	12.99
1963	14.90	12.74	15.20
1964	16.32	13.95	16.04
1965	15.17	13.46	14.71
1966	15.37	13.40	15.17
1967	15.26	13.78	15.20
1968	15.25	14.51	15.70[d]
1969	14.19	13.33	14.52[e]
1970	14.85	14.11	15.66
1971	15.04	13.97	15.01
1972	14.40	13.84	13.39
1973	15.35	13.87	14.41

[a]Estimated from 50, 51, 52 data by method of ratios.
[b]Estimated from 50, 51, 52 data by method of ratios.
[c]Estimated from 50, 51, 52 data by method of ratios.
[d]Estimated from 66, 67, 70, 71 data by method of ratios.
[e]Estimated from 66, 67, 70, 71 data by method of ratios.

the wind turbine to be poor in the first year after installation. This has economic implications because the cash flow requirements will usually be most critical the first year after installation. The turbine may go ahead and deliver the expected amount of energy over its lifetime, but this does not help the economic situation immediately after installation. Justus[14] examined this question also, for 40 sites in the United States, and concluded that there is about a 60 % probability that one low wind speed month or year will be followed by a similar one. If wind speeds were totally random, then this probability would be 50 %, the same as for two heads in a row when flipping a coin. The probability of an additional low speed month or

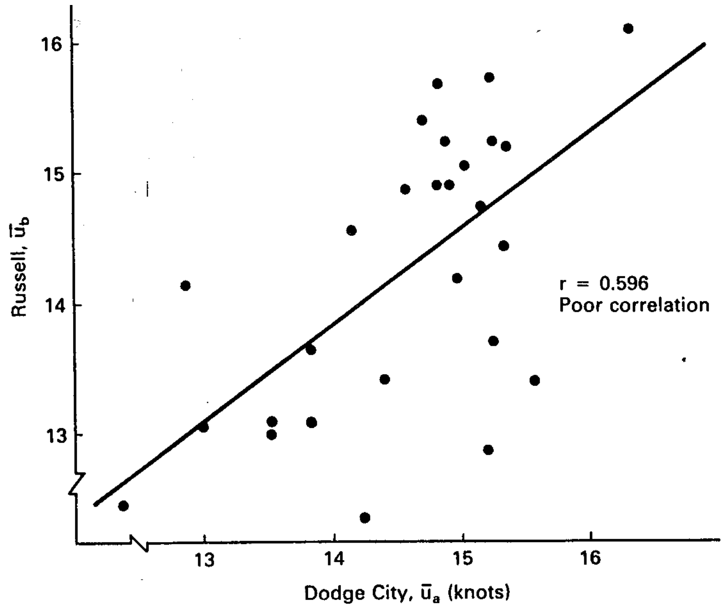

Figure 23: Yearly mean wind speeds (knots) at Russell, Kansas, versus simultaneous speeds at Dodge City, Kansas.

year following the first two is totally random. Based on these results, Justus concluded that clusters of bad wind months or years are not highly probable. In fact, the variation of speed from one period to the next is almost totally random, and therefore such clusters should not be of major concern.

We have seen that the 90 % confidence interval for the long term annual mean speed at a site is between $0.9\bar{u}$ and $1.1\bar{u}$ when only one year's data is collected. If this is not adequate, then additional data need to be collected. Corotis[7] reports that the 90 % confidence interval is between $0.93\bar{u}$ and $1.07\bar{u}$ for two year's data, and between $0.94\bar{u}$ and $1.06\bar{u}$ for three year's data. The confidence interval is basically inversely proportional to the square root of the time period, so additional years of data reduce the confidence interval at a slower and slower rate.

10 DISTRIBUTION OF EXTREME WINDS

Two important wind speeds which affect turbine cost are the design wind speed which the rotor can withstand in a parked rotor configuration, and the maximum operating wind speed. A typical wind turbine may start producing power at 5 to 7 m/s (11 to 16 mi/h), reach rated power at 12 to 16 m/s (27 to 36 mi/h), and be shut down at a maximum operating speed of 20 to 25 m/s (45 to 56 mi/h).

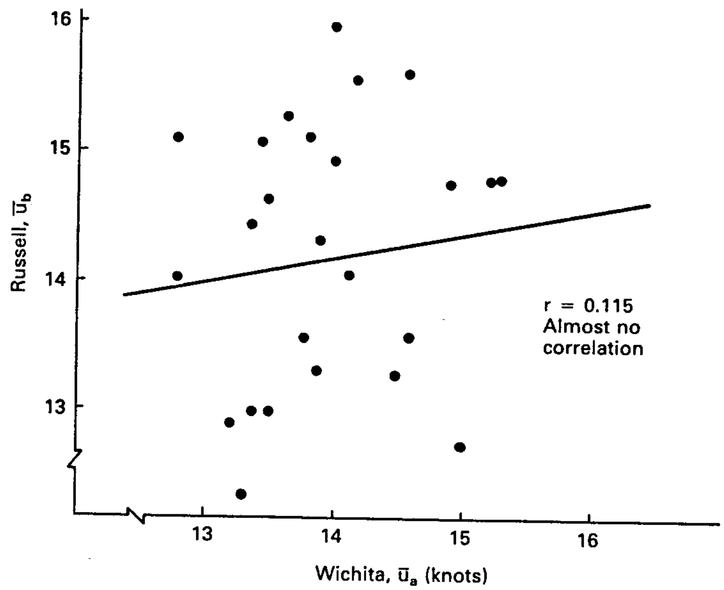

Figure 24: Yearly mean wind speeds (knots) at Russell, Kansas, versus simultaneous speeds at Wichita, Kansas.

The design wind speed is prescribed in a ANSI Standard[1], as modified by height and exposure. This varies from 31 m/s (70 mi/h) to 49 m/s (110 mi/h) in various parts of the United States. It is not uncommon for wind turbines to be designed for 55 m/s (125 mi/h) or more. This high design speed may be necessary at mountainous or coastal sites, but may be unnecessary and uneconomical in several Great Plains states where the highest wind speed ever measured by the National Weather Service is less than 45 m/s (100 mi/h).

Also of interest is the number of times during a year that the wind reaches the maximum turbine operating wind speed so the turbine must be shut down. This affects the turbine design and has an impact on the utility when the power output suddenly drops from rated to zero. For example, if a wind farm had to be shut down during the morning load pickup, other generators on the utility system must ramp up at a greater rate, perhaps causing operational problems.

Wind turbine designers and electric utility operators therefore need accurate models of extreme winds, especially the number of times per year the maximum operating speed is exceeded. Variation of extreme winds with height is also needed. The ANSI Standard[1] gives the extreme wind that can be expected once in 50 years, but daily and monthly extremes are also of interest.

The extreme wind data which are available in the United States are the monthly or yearly fastest mile of wind at a standard anemometer height. Anemometers will be discussed in detail in the next chapter, but the electrical contact type used by the National Weather

Service stations will be mentioned briefly here. Its three-cup rotor drives a gear train which momentarily closes a switch after a fixed number of revolutions. The cup speed is proportional to the wind speed so contact is made with the passage of a fixed amount of air by or through the anemometer. The anemometer can be calibrated so the switch makes contact once with every mile of wind that passes it. If the wind is blowing at 60 mi/h, then a mile of wind will pass the anemometer in one minute.

The contact anemometer is obviously an averaging device. It gives the average speed of the fastest mile, which will be smaller than the average speed of the fastest 1/10th mile, or any other fraction of a mile, because of the fluctuation of wind speed with time. But one has to stop refining data at some point, and the fastest mile seems to have been that point for structural studies.

The procedure is to fit a probability distribution function to the observed high wind speed data. The Weibull distribution function of Eq. 30 could probably be used for this function, except that it tends to zero somewhat too fast. It is convenient to define a new distribution function $F_e(u)$ just for the extreme winds.

Weather related extreme events, such as floods or extreme winds[11], are usually described in terms of one of two Fisher-Tippett distributions, the Type I or Type II. Thom[23, 24] used the Type II to describe extreme winds in the United States. He prepared a series of maps based on annual extremes, showing the fastest mile for recurrence intervals of 2, 50, and 100 years. Height corrections were made by applying the one-seventh power law. National Weather Service data were used.

The ANSI Standard[1] is based on examination of a longer period of record by Simiu[21] and uses the Type I distribution. Only a single map is given, for a recurrence interval of 50 years. Other recurrence intervals are obtained from this map by a multiplying factor. Tables are given for the variation of wind speed with height.

Careful study of a large data set collected by Johnson and analyzed by Henry[12] showed that the Type I distribution is superior to the Type II, so the mathematical description for only the Type I will be discussed here. The Fisher-Tippett Type I distribution has the form

$$F_e(u) = \exp(-\exp(-\alpha(x-\beta))) \tag{68}$$

where $F_e(u)$ is the probability of the annual fastest mile of wind speed being less than u. The parameters α and β are characteristics of the site that must be estimated from the observed data. If n period extremes are available, the maximum likelihood estimate of α may be obtained by choosing an initial guess and iterating

$$\alpha_{i+1} = \frac{1}{\bar{x} - \dfrac{\sum_{j=1}^{n} x_j \exp(-\alpha_i x_j)}{\sum_{j=1}^{n} \exp(-\alpha_i x_j)}} \tag{69}$$

until convergence to some value $\tilde{\alpha}$. The maximum likelihood estimate of β is

$$\beta = \bar{x} - \frac{0.5772}{\tilde{\alpha}} \tag{70}$$

If $F_e(u) = 0.5$, then u is the median annual fastest mile. Half the years will have a faster annual extreme mile and half the years will have a slower one. Statistically, the average time of recurrence of speeds greater than this median value will be two years. Similar arguments can be made to develop a general relationship for the *mean recurrence interval* M_r, which is

$$M_r = \frac{1}{1 - F_e(u)} \tag{71}$$

which yields

$$u = -\frac{1}{\alpha} \ln\left(-\ln(1 - \frac{1}{M_r} \right) + \beta \tag{72}$$

A mean recurrence interval of 50 years would require $F_e(u) = 0.98$, for example.

Example

At a given location, the parameters of Eq. 68 are determined to be $\gamma = 4$ and $\beta = 20$ m/s. What is the mean recurrence interval of a 40 m/s extreme wind speed?

From Eq. 68 we find that

$$F_e(40) = \exp\left[-\left(\frac{40}{20}\right)^{-4} \right] = 0.939$$

From Eq. 67, the mean recurrence interval is

$$M_r = \frac{1}{1 - 0.939} = 16.4$$

At this location, a windspeed of 40 m/s would be expected about once every 16 years, on the average.

If we have 20 years or more of data, then it is most appropriate to find the distribution of yearly extremes. We use one value for each year and calculate α and β. If we have only a few years of data, then we might want to find the distribution of monthly extremes. The procedure is the same, but we now have 12 values for each year instead of one. If we have only a few months to a year of wind data, then we can find the distribution of daily extremes.

When modeling is conducted using one period length and it is desired to express the results in terms of a larger period, the parameters of the distribution must be altered. If there are n of the smaller periods within each larger unit, the distribution function for the larger period is

$$F_L = [F_s]^n \tag{73}$$

where F_s is the distribution function for the smaller period. The relationship between parameters is then

$$\alpha_L = \alpha_s \qquad \text{and} \qquad \beta_L = \beta_s + \frac{\ln n}{\alpha_s} \tag{74}$$

For example, the relationship between monthly and yearly extremes is

$$\alpha_a = \alpha_m, \qquad \beta_a = \beta_m + \frac{2.485}{\alpha_m} \tag{75}$$

An example of values of α and β for daily extremes is shown in Table 2.7. These are calculated from a large data set collected by Kansas State University for the period October, 1983 through September, 1984 at 7 Kansas locations, using the daily fastest minute.

Table 2.7 α and β for seven Kansas sites, daily fastest minute in mi/h

	50 m		30 m		10 m	
	α	β	α	β	α	β
Tuttle Creek Lake	0.1305	23.47	0.1298	22.25	0.1414	20.76
Plainville	0.1410	26.04	0.1474	23.63	0.1660	19.57
Beaumont	0.1441	26.14	0.1522	24.71	0.1620	21.74
Lyndon	0.1582	23.00	0.1547	21.24	0.1648	18.21
Dodge City	0.1448	25.86	0.1483	23.92	0.1552	20.83
Lyons	0.1537	23.55	0.1594	21.75	0.1689	19.49
Oakley	0.1435	26.89	0.1457	24.51	0.1556	20.72
mean	0.1451	24.24	0.1482	23.14	0.1591	20.19
std. dev.	0.0089	2.16	0.0094	1.39	0.0094	1.17

Differences among sites are relatively small, as seen by the standard deviations. There is a tendency for α to get smaller and β to get larger with height.

Similar data for 5 Kansas National Weather Service Stations for the same time period is shown in Table 2.8. The fastest mile is used rather than the fastest minute, which theoretically makes only a small difference.

Table 2.8 α and β for five Kansas NWS Stations, daily fastest mile.

	α	β	Height in ft
Wichita	0.2051	17.01	25
Topeka	0.2068	14.89	20
Concordia	0.2149	17.45	33
Goodland	0.2272	18.20	20
Dodge City	0.2060	19.46	20
mean	0.2120	17.40	
std. dev.	0.0094	1.68	

Differences among the NWS sites are also relatively small. However, the NWS sites have a consistently larger α and smaller β than the KSU sites in Table 2.7. Both differences cause the predicted extreme wind to be smaller at the NWS sites than at the KSU sites. In making a detailed comparison of the Dodge City NWS and the Dodge City KSU sites, which are about 5 miles apart in similar terrain, it appears that the difference was caused by the type of anemometer used. The KSU study used the Maximum anemometer, a small, light, and inexpensive device, while the National Weather Service used the Electric Speed Indicator anemometer, a much larger and heavier device. The 10 m mean speed with the KSU equipment was 12.8 mi/h for the year, while the NWS mean was 14.1 mi/h, so the Maximum anemometer indicated a mean speed 9 % below the NWS mean. On the other hand, the extreme winds measured by the Maximum anemometer averaged 18 % higher than the NWS extremes. The Maximum anemometer indication for the extreme winds varied from about the same as the NWS measurement to as much as 15 mi/h faster. It is evident that the type of measurement equipment can make a significant difference in the results.

A longer period of time was then analyzed for two of the NWS Stations, using monthly extremes rather than daily extremes. The results of this study are shown in Table 2.9.

Table 2.9. α and β for Topeka and Dodge City, using monthly fastest mile for 1949–1968.

	α	β
Topeka	0.1420	34.38
Dodge City	0.1397	41.94

The fastest mile observed at Dodge City for this period was 77 mi/h, while at Topeka it was 74 mi/h. The α values are much closer to the KSU values than to the NWS values for the one year study. The β values are larger as required by Eq. 74. From these values for Dodge City, the extreme wind expected once every 3.3 months (approximately 100 days) would be 49.23 mi/h, a value surprisingly close to the prediction using the KSU data and daily extremes.

As mentioned earlier, it is important to have an estimate of the number of days during a year in which the wind speed should equal or exceed a given value. This is done by inserting values of α, β, and the given wind speed in Eq. 68, then using Eq. 69 to find M_r, and finally taking the reciprocal of the interval M_r to find the frequency. Values are given in Table 2.10 for the mean KSU parameters, the mean NWS parameters from Table 2.8, and the mean NWS parameters for monthly extremes from Table 2.9, all expressed in days per year for a wind speed in mi/h.

Table 2.10. Days per year a given wind speed or greater is observed.

speed	50 m	30 m	10 m	NWS	Topeka	Dodge City
30	128.4	110.8	69.1	39.3		
35	69.1	57.8	33.0	14.6		
40	35.3	28.8	15.3	5.3	4.4	8.8
45	17.5	14.0	7.0	1.9	2.4	5.8
50	8.6	6.8	3.2	0.7	1.2	3.3
55	4.2	3.2	1.4	0.2	0.6	1.8
60	2.0	1.6	0.6	0.1	0.3	0.9

There is some scatter in the results, as has been previously discussed, but the results are still quite acceptable for many purposes. For example, a wind turbine with a maximum operating speed of 45 mi/h and a height of 10 m or less would be expected to be shut down between 2 and 7 times per year. If the turbine height were increased to 30 m, the number of shutdowns would approximately double, from 7 to 14. Increasing the maximum operating speed by 5 mi/h will cut the number of shutdowns in half. These rules of thumb may be quite useful to system designers.

As mentioned earlier, Thom[23] has analyzed 141 open-country stations averaging about 15 years of data per station and has published results for $F_e(u) = 0.5$, 0.98, and 0.99 which correspond to mean times between occurrences of 2, 50, and 100 years, respectively. These curves are given in Figs. 25–27. Since he used the Type II distribution, the absolute values are not as accurate as we get from the Type I. However, the trends are the same with either distribution and illustrate some very interesting facts about the geographical variation of extreme winds.

The curve for the 2 year mean recurrence interval gives results which might be anticipated from information on average winds presented earlier in the chapter. The High Plains have the highest extreme wind speeds and the southeastern and southwestern United States have the lowest. This relationship is basically the same as that for the average wind speeds. The 50 year and 100 year curves are distinctly different, however. The *once a century* extreme wind in Western Kansas, Oklahoma, and Texas is about 40 m/s, a figure exceeded in large areas of the coastal regions. The Gulf Coast and Atlantic Coast, as far north as Southern Maine, have experienced extreme winds from tropical cyclones, or hurricanes. The effect of these storms often extends inland from 100 to 200 miles. The remainder of the United States experiences extreme winds largely from thunderstorms. This type of storm accounts for over one third of the extreme-wind situations in the contiguous United States.

Water areas have a marked effect on extreme wind speeds. Where a location has unobstructed access to a large body of water, extreme winds may be 15 m/s or more greater than a short distance inland. High winds in cyclones and near water tend to remain steady over longer periods than for thunderstorms. They also tend to be much more widespread in a given situation and hence cause widespread damage, although damage to individual structures may not be greater than from thunderstorm winds.

The speed of the greatest gust experienced by a wind turbine will be somewhat greater than the speed of the fastest mile because of the averaging which occurs during the period of measurement. This will vary with the period chosen for the gust measurement and on the speed of the fastest mile. It appears that a good estimate of a three second gust speed is 1.25 times the speed of the fastest mile[15]. That is, if the fastest mile is measured at 40 m/s, the fastest gust will probably be about 1.25(40) = 50 m/s. This is the speed which should be used in talking about survivability of wind turbines.

Standard civil engineering practice calls for ordinary buildings to be designed for a 50 year recurrence interval, and for structures whose collapse do not threaten human safety to be designed for a 25 year recurrence interval. The civil engineers then typically add safety factors to their designs which cause the structures to actually withstand higher winds. A similar approach would seem appropriate for wind turbines[15]. A 50 year recurrence interval would imply that a design for a maximum gust speed of 50 m/s would be adequate over most of the United States, with perhaps 65 m/s being desirable in hurricane prone areas. Wind turbine costs tend to increase rapidly as the design speed is increased, so rather careful cost studies are required to insure that the turbine is not over designed. Wind turbines are different from public buildings and bridges in that they would normally fail in high winds without people getting hurt. It may be less expensive to replace an occasional wind turbine with a design speed of 50 m/s than to build all wind turbines to withstand 65 m/s.

11 PROBLEMS

1. What is the density of dry air in kg/m^3 at standard pressure (101.3 kPa) and
   (a) T = 35^{o}C
   (b) T = -25^{o}C?

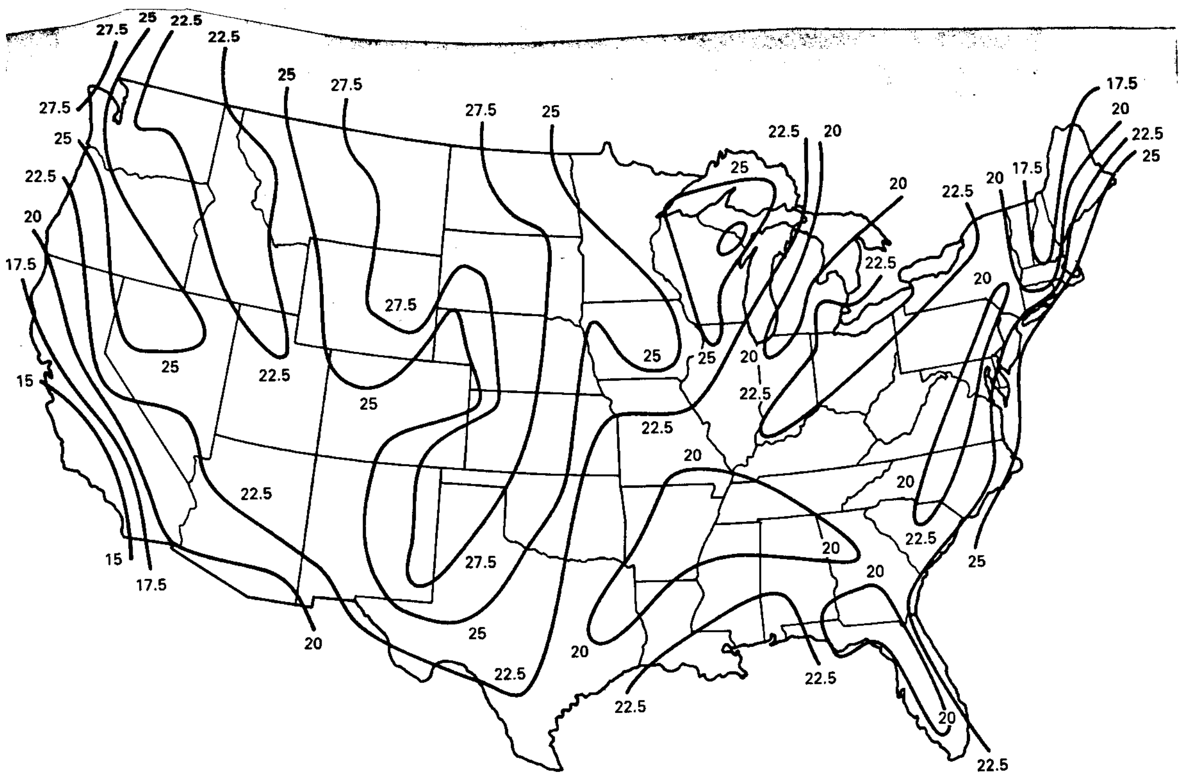

Figure 25: Isotach 0.50 quantiles in meters per second. Annual extreme mile (1.6km) 10 m above ground; 2-year mean recurrence interval. (After [23].)

2. Locate several of the world's major deserts on a map. List at least five of these deserts with an estimate of the range of latitude. (For example, the American Southwest extends from about 25^oN Latitude to about 37^oN Latitude.) Compare with Fig. 5.

3. What is the nominal air pressure in kPa as predicted by the U. S. Standard Atmosphere curve at

   (a) Dodge City, elevation 760 m.

   (b) Denver, elevation 1600 m.

   (c) Pike's Peak, elevation 4300 m.

4. If the temperature at ground level is 30^oC, at what altitude would you reach the freezing point of water in an atmosphere where the lapse rate is adiabatic?

5. Dodge City is at an elevation of 760 m above sea level. A parcel of air 1500 m above Dodge City (2260 m above sea level) descends adiabatically to ground level. If its initial temperature is 0^oC, what is its final temperature?

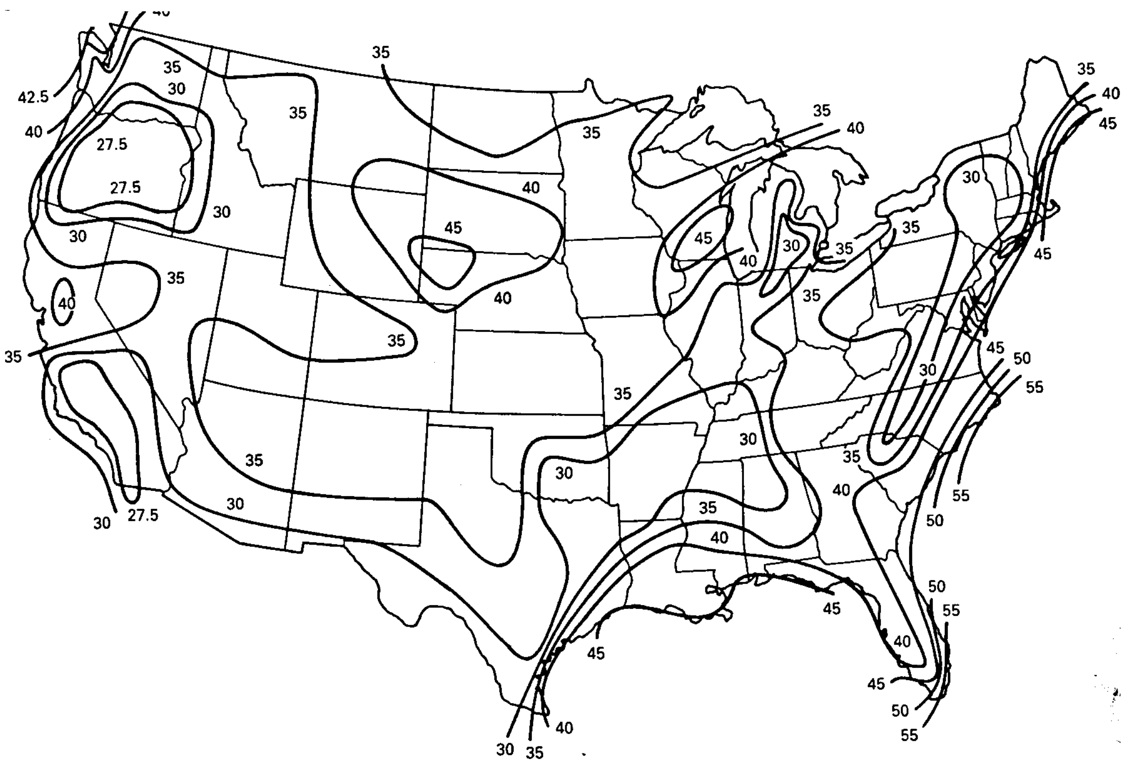

Figure 26: Isotach 0.02 quantiles in meters per second. Annual extreme mile (1.6 km) 10 m above ground; 50-year mean recurrence interval. (After [23].)

6. Ten measured wind speeds are 5, 9, 6, 8, 12, 7, 8, 5, 11, and 10 m/s. Find the mean, the variance, the standard deviation, and the median speed.

7. A wind data acquisition system located at Kahuku Point, Hawaii, measures 7 m/s 24 times, 8 m/s 72 times, 9 m/s 85 times, 10 m/s 48 times, and 11 m/s 9 times during a given period. Find the mean, variance, and standard deviation.

8. For the data of the previous problem, find the probability of each wind speed being observed. Compute the cumulative distribution function $F(u_i)$ for each wind speed. What is the probability that the wind speed will be 10 m/s or greater?

9. Evaluate $f(u)$ of Eq. 30 for $c = 1$ for $u = 0.1, 0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6,$ and 1.8 if

   (a) $k = 3.2$
   (b) $k = 0.8$
   (c) Plot $f(u)$ versus u in a plot similar to Fig. 16.

10. Evaluate the gamma function of Eq. 35 to five significant places for

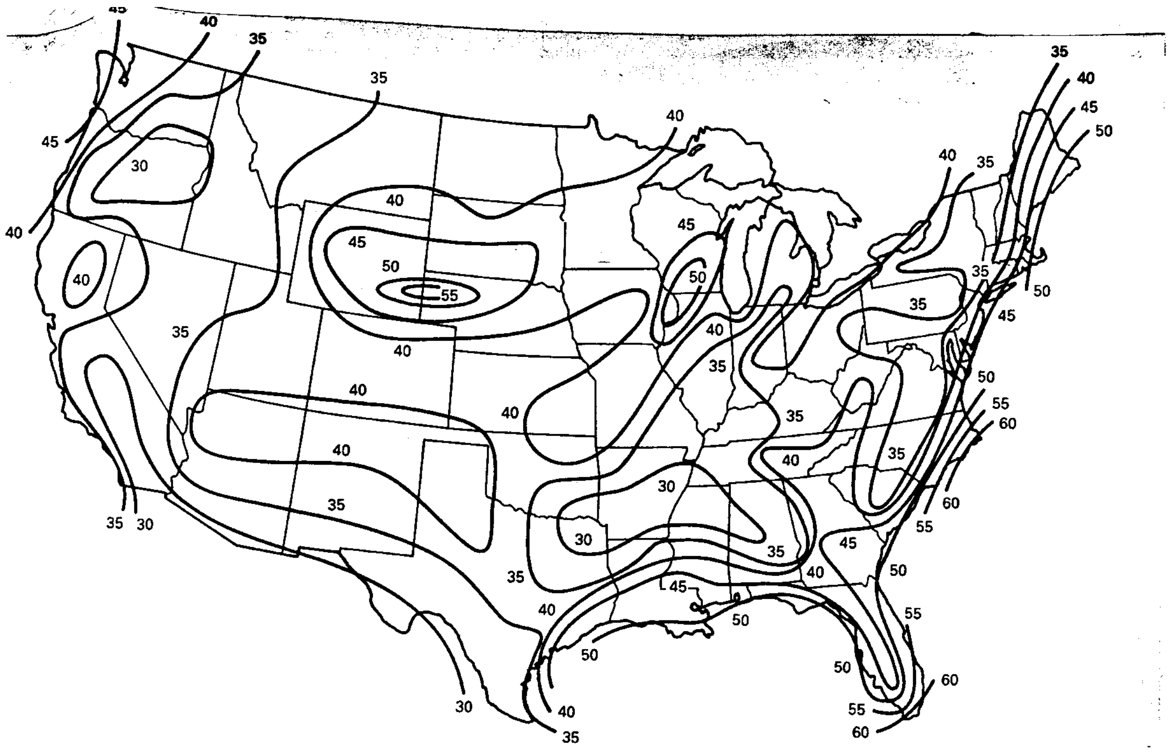

Figure 27: Isotach 0.01 quantiles in meters per second. Annual extreme mile (1.6 km) 10 m above ground; 100-year mean recurrence intervals. (After [23].)

(a) $k = 1.2$

(b) $k = 2.8$

Note: If you use the math tables, you will need to use linear interpolation to get five significant places.

11. Values of wind speed u_i in knots and numbers of readings per year m_i at each speed u_i are given below for Dodge City in 1971. Note that there are 2920 readings for that year.

    (a) Compute the mean wind speed in knots. Express it in m/s.

    (b) Prepare a table of u_i, x_i, m_i, $f(u_i)$, $F(u_i)$, and y similar to Table 2.3.

    (c) Plot the data points of y versus x similar to Fig. 18.

    (d) Ignore wind speeds below 3 knots and above 31 knots and compute $\bar{x}$, $\bar{y}$, a, b, k, and c using Eqs. 55– 56.

u_i	1	2	3	4	5	6	7	8	9	10
m_i	82	5	69	81	137	199	240	274	157	291
u_i	11	12	13	14	15	16	17	18	19	20
m_i	134	186	167	140	159	110	124	141	38	67
u_i	21	22	23	24	25	26	27	28	29	30
m_i	16	21	21	17	12	7	5	5	0	7
u_i	31	32	33	34	35	36	37	38	39	40
m_i	0	3	1	1	3	0	0	0	0	0

12. At a given site, the mean wind speed for a month is 6 m/s with a standard deviation of 2.68 m/s. Estimate the Weibull parameters k and c using the Justus approximation of Eq. 49.

13. Weibull parameters at a given site are $c = 7$ m/s and $k = 2.6$. About how many hours per year will the wind speed be between 8.5 and 9.5 m/s? About how many hours per year will the wind speed be greater than 10 m/s? About how many hours per year will the wind speed be greater than 20 m/s?

14. The Dodge City anemometer was located 17.7 m above the ground for the 14 year period 1948-61. The location was at the Dodge City airport, on top of a gentle hill, and with excellent exposure in all directions. The average Weibull parameters for this period were $c = 14.87$ knots and $k = 2.42$. The mean speed was 12.91 knots.

    (a) Plot both the Weibull and Rayleigh probability density functions $f(u)$ on the same sheet of graph paper.

    (b) Find the probability $P(u \geq u_a)$ of the wind speed being greater than or equal to 10, 15, and 20 knots for both distributions.

    (c) How does the Rayleigh density function compare with the Weibull over the ranges 0-10, 11-20, and over 20 knots?

15. The monthly mean speeds for July in Dodge City for the years 1948 through 1973 are 12.82, 10.14, 10.49, 10.86, 14.30, 13.13, 12.13, 13.38, 11.29, 12.10, 12.56, 9.03, 12.20, 9.18, 9.29, 12.02, 12.01, 10.21, 11.64, 9.56, 10.10, 10.06, 11.38, 10.19, 10.14, and 10.50 knots.

    (a) Find the mean $\bar{u}_m$ and the normalized standard deviation $\sigma_m/\bar{u}_m$.

    (b) Find the 90% confidence interval for $\bar{u}_m$, using the actual standard deviation. How many months are outside this interval?

    (c) Find the 90% confidence interval for the best and worst months, using the 90 percentile standard deviation for all U. S. sites. Do these intervals include the long term monthly mean?

16. The yearly mean speeds for Dodge City for the years 1948 through 1973 are 13.50, 12.15, 11.54, 12.41, 12.77, 14.04, 13.68, 13.34, 13.12, 13.23, 12.42, 13.41, 13.71, 11.05, 9.93, 11.06, 12.29, 11.31, 11.48, 11.40, 11.37, 10.48, 11.04, 11.22, 10.65, and 11.44 knots.

    (a) Find the mean $\bar{u}$ and the standard deviation σ.

    (b) Find the 90 % confidence interval for $\bar{u}$, using the actual standard deviation. How many years are outside this confidence interval?

    (c) Discuss the accuracy of using just the last year and a 50 percentile standard deviation to estimate the long term mean.

17. Find the equation for Wichita wind speeds versus Dodge City wind speeds for the data in Table 2.6. What is the correlation coefficient r? (Note: This requires a hand calculator with linear regression capability or access to a computer.)

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