Measurement of wind speed is very important to people such as pilots, sailors, and farmers. Accurate information about wind speed is important in determining the best sites for wind turbines. Wind speeds must also be measured by those concerned about dispersion of airborne pollutants.

Wind speeds are measured in a wide variety of ways, ranging from simple go-no go tests to the most sophisticated electronic systems. The variability of the wind makes accurate measurements difficult, so rather expensive equipment is often required.

Wind direction is also an important item of information, as well as the correlation between speed and direction. In the good wind regime of the western Great Plains, prevailing winds are from the north and south. Winds from east and west are less frequent and also have lower average speeds than the winds from north and south. In mountain passes, the prevailing wind direction will be oriented with the pass. It is conceivable that the most economical wind turbine for some locations will be one that is fixed in direction so that it does not need to turn into the wind. If energy output is not substantially reduced by eliminating changes in turbine orientation, then the economic viability of that wind turbine has been improved. But we must have good data on wind direction before such a choice can be made.

We shall examine several of the methods of measuring both wind speed and direction in this chapter.

1 EOLIAN FEATURES

The most obvious way of measuring the wind is to install appropriate instruments and collect data for a period of time. This requires both money and time, which makes it desirable to use any information which may already be available in the surface of the earth, at least for preliminary investigations. The surface of the earth itself will be shaped by persistent strong winds, with the results called eolian features or eolian landforms[5]. These eolian landforms are present over much of the world. They form on any land surface where the climate is windy. The effects are most pronounced where the climate is most severe and the winds are the strongest. An important use of eolian features will be to pinpoint the very best wind energy sites, as based on very long term data.

Sand dunes are the best known eolian feature. Dunes tend to be elongated parallel to the dominant wind flow. The wind tends to pick up the finer materials where the wind speed is higher and deposit them where the wind speed is lower. The size distribution of sand at a
given site thus gives an indication of average wind speed, with the coarser sands indicating higher wind speeds.

The movement of a sand dune over a period of several years is proportional to the average wind speed. This movement is easily recorded by satellite or aerial photographs.

Another eolian feature is the \textit{playa lake}. The wind scours out a depression in the ground which fills with water after a rain. When the water evaporates, the wind will scour out any sediment in the bottom. These lakes go through a maturing process and their stage of maturity gives a relative measure of the strength of the wind.

Other eolian features include \textit{sediment plumes} from dry lakes and streams, and \textit{wind scour}, where airborne materials gouge out streaks in exposed rock surfaces.

Eolian features do not give precise estimates for the average wind speed at a given site, but can identify the best site in a given region for further study. They show that moving a few hundred meters can make a substantial difference in a wind turbine output where one would normally think one spot was as good as another. We can expect to see substantial development of this measurement method over the next few decades.

\section{2 BIOLOGICAL INDICATORS}

Living plants will indicate the effects of strong winds as well as eolian features on the earth itself. Eolian features are most obvious where there is little plant cover, so the plants which hide eolian features may be used for wind information instead. Strong winds deform trees and shrubs so that they indicate an integrated record of the local wind speeds during their lives. The effect shows up best on coniferous evergreens because their appearance to the wind remains relatively constant during the year. Deciduous trees shed their leaves in the winter and thus change the exposed area tremendously. If the average wind speed is high but still below some critical value, above which deciduous trees cannot survive, they will not indicate relative differences in wind speeds very well, although they do show distinctive wind damage.

Putnam lists five types of deformation in trees: \textit{brushing}, \textit{flagging}, \textit{throwing}, \textit{wind clipping}, and \textit{tree carpets}[8]. A tree is said to be \textit{brushed} when the branches are bent to leeward (downwind) like the hair in an animal pelt which has been brushed one way. Brushing is usually observable only on deciduous trees and then only when the leaves are off. It will occur with light prevailing winds, and is therefore of little use as a wind prospecting tool.

A tree is said to be \textit{flagged} when the wind has caused its branches to stretch out to leeward, perhaps leaving the windward side bare, so the tree appears like a flagpole carrying a banner in the breeze. This is an easily observed and measured effect which occurs over a range of wind speeds important to wind power applications.

A tree is said to be \textit{windthrown} when the main trunk, as well as the branches, is deformed so as to lean away from the prevailing wind. This effect is produced by the same mechanism
which causes flagging, except that the wind is now strong enough to modify the growth of the upright leaders of the tree as well as the branches.

Trees are said to be wind clipped when the wind has been sufficiently severe to suppress the leaders and hold the tree tops to a common, abnormally low level. Every twig which rises above that level is promptly killed, so that the upper surface is as smooth as a well-kept hedge.

Tree carpets are the extreme case of clipping in that a tree may grow only a few centimeters tall before being clipped. The branches will grow out along the surface of the ground, appearing like carpet because of the clipping action. The result may be a tree 10 cm tall but extending 30 m to leeward of the sheltering rock where the tree sprouted.

Hewson and Wade have proposed a rating scale\cite{2} for tree deformation which is shown in Fig. 1. In this scale 0 corresponds to no wind damage, I, II, III, and IV to various degrees of flagging, V to flagging plus clipping, and VI to throwing. Class VII is a flagged tree with the flagging caused by other factors besides a strong prevailing wind, such as salt spray from the ocean or mechanical damage from a short, intense storm.

The average wind speed at which these effects occur will vary from one species to another so calibration is necessary. This is a long, laborious process so refinements can be expected for a number of years as new data are reported. Putnam has reported calibration data for balsam trees in New England\cite{8}, which are given in Table 3.1. The wind velocity at the top of the specimen is given. This must be translated to wind speed at wind turbine hub height by the shear equations found in the previous chapter, when wind turbine power estimates are required.

It can be seen in Table 3.1 that the full range of observed deformations occur over the speed range of 7 to 12 m/s, which is an important range of interest to wind turbines. The balsam trees can be used to effectively rank various sites and to eliminate many marginal locations.

Deformations also show the location of high wind speed zones produced by standing waves or gravity waves in the air flow over rough surfaces. A high wind striking a mountain top may be deflected downward and hit in the valley or on the side of a hill with much greater force than the expected prevailing wind at that altitude. Putnam mentions the Wamsutta Ridge of Mt. Washington as a good example of this effect. Along most of the crest of the ridge the trees grow between 5 and 10 m high. But in one patch, a hundred meters wide, the high speed upper level winds have been deflected downward and sear the ridge. Here balsam grows only in the lee of rocks and only to a height of 0.3 m. The transition from the high wind zone to the normal wind zone occurs in a matter of meters. Finding such a zone is to the wind prospector what finding gold is to the miner.

Once such zones have been identified, it is still important to place wind instrumentation at those sites. The eolian and biological indicators help identify the very best sites and eliminate the poor sites without giving the precise wind data needed for wind turbine deployment.
Figure 1: Representation of the rating scale based on the shape of the crown and degree of bending of twigs, branches, and the trunk. Class VII is pure mechanical damage. (From Ref. 3.)

Table 3.1 Putnam’s Calibration of Balsam Deformation versus Average Wind Speed in New England

<table>
<thead>
<tr>
<th>Deformation</th>
<th>Wind Speed $u_x$ at Height $x$ (m) at Top of Specimen above Ground (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balsam held to 0.3 m</td>
<td>$u_{0.3} = 12$ m/s</td>
</tr>
<tr>
<td>Balsam held to 1.2 m</td>
<td>$u_{1.2} = 9.6$ m/s</td>
</tr>
<tr>
<td>Balsam thrown</td>
<td>$u_8 = 8.6$ m/s</td>
</tr>
<tr>
<td>Balsam strongly flagged</td>
<td>$u_9 = 8.3$ m/s</td>
</tr>
<tr>
<td>Balsam flagged</td>
<td>$u_9 = 8.0$ m/s</td>
</tr>
<tr>
<td>Balsam minimally flagged</td>
<td>$u_{12} = 7.7$ m/s</td>
</tr>
<tr>
<td>Balsam unflagged</td>
<td>$u_{12} = 6.9$ m/s</td>
</tr>
</tbody>
</table>
3 ROTATIONAL ANEMOMETERS

Anemometers, instruments that measure wind speed, have been designed in great variety [6, 1]. Each type has advantages and disadvantages, as we shall see. Anemometer types include the propeller, cup, pressure plate, pressure tube, hot wire, Doppler acoustic radar, and laser. The propeller and cup anemometers depend on rotation of a small turbine for their output, while the others basically have no moving parts.

Figure 2 shows a propeller type anemometer made by Weathertronics. This particular model includes both speed and direction in the same sensor. The propeller is made of aluminum and the tail is made of fiberglass. The sensor is able to withstand wind speeds of 90 m/s.

Anemometers may have an output voltage, either dc or ac, or a string of pulses whose frequency is proportional to anemometer speed. The dc generator is perhaps the oldest type and is still widely used. It requires no external power source and is conveniently coupled to a simple dc voltmeter for visual readout or to an analog-to-digital converter for digital use. The major disadvantages are the brushes required on the generator, which must be periodically maintained, and the susceptibility to noise in a recording system dependent on voltage level. The long runs of cable from an anemometer to a recording station make electromagnetic
interference a real possibility also. Anemometers with permanent magnet ac generators in them do not require brushes. However, the ac voltage normally needs to be rectified and filtered before being used. This is difficult to accomplish accurately at the low voltages and frequencies associated with low wind speeds. This type of anemometer would not be used, therefore, where wind speeds of below 5 m/s are of primary interest.

The digital anemometer uses a slotted disk, a light emitting diode (LED), and a photo-transistor to obtain a pulse train of constant amplitude pulses with frequency proportional to anemometer angular velocity. Wind speed can be determined either by counting pulses in a fixed time period to get frequency, or by measuring the duration of a single pulse. In either case, the noise immunity of the digital system is much better than the analog system. The major disadvantages of the digital anemometer would be the complexity, and the power consumption of the LED in battery powered applications. One LED may easily draw more current than the remainder of a data acquisition system. This would not be a consideration where commercial power is available, of course.

Fig. 3 shows a cup type anemometer made by Electric Speed Indicator Company. This is the type used by most National Weather Service stations and airports. The cups are somewhat cone shaped rather than hemispherical and are about 11 cm in diameter. The turning radius of the tip of the cup assembly is about 22 cm. The cups turn a small dc generator which has a voltage output proportional to wind speed. The proportionality constant is such that for every 11.2 m/s increase in wind speed, the voltage increases by 1 volt. Wind speeds below 1 m/s do not turn the cups so there is an offset in the curve of voltage versus wind speed, as shown in Fig. 4. Since the straight line intercepts the abscissa at 1 m/s, an output voltage of 1 V is actually reached at 12.2 m/s.

In equation form, the wind speed \( u \) is given by

\[
u = 11.2V + 1 \quad \text{m/s}
\]

where \( V \) is the output voltage.

A visual indication of wind speed is obtained by connecting this dc generator to a dc voltmeter with an appropriately calibrated scale. The scale needs to be arranged such that the pointer indicates a speed of 1 m/s when the generator is stalled and the voltage is zero. Then any wind speeds above 1 m/s will be correctly displayed if the scale is calibrated according to Fig. 4.

Many applications of anemometers today require a digital output for data collection. This is usually accomplished by an analog-to-digital (A/D) converter. This is an electronic device which converts an analog voltage into a digital number. This digital number may be eight bits long, which gives a maximum range of \( 2^8 \) or 256 different values. It may also be twelve or sixteen bits long which gives more resolution. The cost of this greater resolution may not be justified because of the variability of the wind and the difficulty in measuring it.

The output of a typical 8-bit A/D converter is shown in Fig. 5. The rated voltage \( V_R \) is
divided into 256 increments of $\Delta V$ volts each. The analog voltage appears along the horizontal axis, and the corresponding digital numbers, ranging from 0 to 255, appear along the vertical axis. In this figure, digital 0 represents the first half increment of voltage or from 0 to $AV/2$ volts. The digital 1 then is centered at $\Delta V$ volts and represents the analog voltages between $AV/2$ and $3AV/2$ volts. The digital number 255 represents the voltages between $254.5\Delta V$ and $256\Delta V = V_R$. If the input voltage is above the rated voltage, the A/D converter will put out a signal indicating that the output is not valid. If $V_R$ is 5 volts, which is a typical value, the A/D converter will not have its rating exceeded for wind speeds below 57 m/s for the anemometer of Fig. 3, which is quite adequate for most sites.

Since each digital number represents a range of voltage, it also represents a range of wind speed. This range would be about 0.22 m/s for an 8-bit, 5 volt A/D converter attached to the anemometer of Fig. 3. Wind power computations performed on a digital computer require a single wind speed to represent this range, which is usually selected at the midpoint. This introduces an error because of the cubic variation of wind power with wind speed and because the wind does not usually blow equally at speeds above and below the midpoint, but rather according to some probability density function. This error becomes smaller as the range of speeds represented by a single number becomes smaller. The magnitude of the error is somewhat difficult to determine, but would be well within acceptable limits for a range of 0.22 m/s.

**Example**

An 8-bit A/D converter with the characteristics shown in Fig. 5 is connected to an anemometer with an output voltage characteristic like Fig. 4. The A/D converter has an output digital number
of 47 for a given wind speed. What is the possible range of wind speeds that this digital number represents?

From Fig. 5 we see that a digital value of 47 corresponds to an analog voltage of $47 \Delta V = 47(5/256) = 0.9180$ V. We get the same digital value over a voltage range of $\pm \Delta V/2$, so the voltage range is $0.9180 \pm 0.0098$ or from 0.9082 to 0.9278 V. From Eq. 1 we find that the wind speed at the low end of this range is

$$u = 11.2(0.9082) + 1 = 11.17 \text{ m/s}$$

while at the upper end of this range it is

$$u = 11.2(0.9278) + 1 = 11.39 \text{ m/s}$$

Therefore, the digital value 47 represents a wind speed of $11.28 \pm 0.11$ m/s.

Both cup and propeller anemometers have inertia and require a certain amount of time to accelerate to the new angular velocity when the wind speed increases. This time can be determined by solving the equation of motion for the anemometer. This equation is found by setting the product of the moment of inertia $I$ in kg·m² and the angular acceleration $\alpha$ in rad/rms² equal to the sum of the moments or torques around the anemometer shaft.

$$I\alpha = I \frac{d\omega}{dt} = T_u - T_{bf} - T_{af} \text{ N·m}$$

These torques are illustrated in Fig. 6. $T_u$ is the torque due to the wind speed $u$, and is the driving or forcing function. $T_{bf}$ is the countertorque due to bearing friction. $T_{af}$ is the torque due to air drag or air friction, and $\omega$ is the mechanical angular velocity in radians per second.

If the torques can be written as linear functions of $\omega$, then we have a first order linear
differential equation which is solved easily. Unfortunately the air friction torque is a nonlinear function of $\omega$, being described by\[4\]

$$T_{af} = a_0 \omega^2 + a_1 \omega + a_2$$

where $a_0$, $a_1$, and $a_2$ are constants determined from wind tunnel tests. The driving torque $T_u$ is a nonlinear function of wind speed and also varies with $\omega$. This makes Eq. 2 very difficult to solve exactly. Instead of this exact solution we shall seek analytic insights which might be available from a simpler and less precise solution.

A linearized equation of motion which is approximately valid for small variations in $u$ and $\omega$ about some equilibrium values $u_o$ and $\omega_o$ is given by\[4\]

$$I \frac{d\omega}{dt} + (K_{bf} + K_1 u_o) \omega = K_1 \omega_o u$$

In this equation, the torques of Eq. 2 have been assumed to be represented by

$$T_u = K_1 \omega_o u$$

$$T_{af} = K_1 u_o \omega$$

$$T_{bf} = K_{bf} \omega$$

If the torque due to the bearing friction is small, then at equilibrium $T_u$ and $T_{af}$ have to be numerically equal, as is evident from Eqs. 2 and 6.
The solution to Eq. 4 for a step change in wind speed from $u_o$ to $u_1$ is

$$\omega = \frac{\omega_o}{K_{bf} + K_1 u_o} \left( K_1 u_1 + [K_{bf} + K_1 (u_o - u_1)] e^{-t/\tau} \right) \text{ rad/s} \quad (6)$$

The time constant $\tau$ is given by

$$\tau = \frac{I}{K_{bf} + K_1 u_o} \text{ s} \quad (7)$$

For good bearings, the bearing friction torque is small compared with the aerodynamic torques and can be neglected. This simplifies the last two equations to the forms

$$\omega = \omega_o \frac{u_1}{u_o} + (u_o - u_1) \frac{\omega_o}{u_o} e^{-t/\tau} \text{ rad/s} \quad (8)$$

$$\tau = \frac{I}{K_1 u_o} \text{ s} \quad (9)$$

Equation 8 shows that the angular velocity of a linearized anemometer is directly proportional to the wind speed when transients have disappeared. Actual commercial anemometers satisfy this condition quite well. The transient term shows an exponential change in angular velocity from the equilibrium to the final value. This also describes actual instrumentation rather well, so Eqs. 8 and 9 are considered acceptable descriptors of anemometer performance even though several approximations are involved.

A plot of $\omega$ following a step change in wind speed is given in Fig. 7. The angular velocity increases by a factor of $1 - 1/e$ or 0.63 of the total increase in one time constant $\tau$. In the
next time constant, $\tau$ increases by 0.63 of the amount remaining, and so on until it ultimately reaches its final value.

During the transient period the anemometer will indicate a wind speed $u_i$ different from the actual ambient wind speed $u$. This indicated wind speed will be proportional to the anemometer angular velocity $\omega$.

$$u_i = K_i \omega$$  \hspace{1cm} (10)

For a cup-type anemometer, the tip speed of the cups will be approximately equal to the wind speed, which means that $K_i$ would be approximately the radius of rotation of the cup assembly. Propeller type anemometers will have a higher tip speed to wind speed ratio, perhaps up to a factor of five or six, which means that $K_i$ would be proportionately smaller for this type of anemometer.

At equilibrium, $u_i = u_o$ and $\omega = \omega_o$, so

$$u_o = K_i \omega_o$$  \hspace{1cm} (11)

When Eqs. 10 and 11 are substituted into Eq. 8 we get an expression for the indicated wind speed $u_i$:

$$u_i = K_i \omega = K_i \left[ \frac{\omega_o u_i}{K_i \omega_o} + (u_o - u_1) \frac{\omega_o}{K_i \omega_o} e^{-t/\tau} \right]$$
\[ u_i = u_1 + (u_o - u_1)e^{-t/\tau} \quad (12) \]

If the indicated wind speed would happen not to be linearly proportional to \( \omega \), then a more complicated expression would result.

The time constant of Eq. 9 is inversely proportional to the equilibrium wind speed. That is, when the equilibrium speed increases, the anemometer will make the transition to its final speed more rapidly for a given difference between equilibrium and final speeds. This means that what we have called a time constant is not really a constant at all. It is convenient to multiply \( \tau \) by \( u_o \) in order to get a true constant. The product of time and speed is distance, so we define a distance constant as

\[ d_m = u_o \tau = \frac{I}{K_1} \text{ m} \quad (13) \]

The distance constant is 7.9 m for the Electric Speed Indicator anemometer shown in Fig. 3. This means that 7.9 m of air must pass by the anemometer before its speed will change by 63 per cent of the difference in the old and new wind speeds. Most commercially available anemometers have distance constants in the range of 1 to 8 m, with the smaller numbers associated with smaller, lighter anemometers[7]. The smaller anemometers are better able to follow the high frequency variations in wind speed in the form of small gusts. A gust of 1 m diameter can easily be observed with an anemometer whose \( d_m = 1 \) m, but would hardly be noticed by one whose \( d_m = 8 \) m. Wind turbines will have a slower response to changes in wind speed than even the slowest anemometer, so knowledge of the higher frequency components of wind speed is not of critical importance in wind power studies. The smaller anemometers are more commonly used in studies on evaporation and air pollution where the smaller gusts are important and the wind speeds of interest are less than perhaps 8 m/s.

**Example**

An Electric Speed Indicator anemometer is connected to a data acquisition system which samples the wind speed each 0.2 s. The distance constant is 7.9 m. Equilibrium has been reached at a wind speed of 5 m/s when the wind speed increases suddenly to 8 m/s. Make a table of wind speeds which would be recorded by the data acquisition system during the first second after the step increase of wind speed.

From Eq. 13, the time constant for an initial wind speed of 5 m/s is

\[ \tau = \frac{d_m}{u_o} = \frac{7.9}{5} = 1.58 \text{ s} \]

From Eq. 12 the indicated wind speed is

\[ u_i = 8 - 3e^{-t/1.58} \text{ m/s} \]

The following table for indicated wind speed is obtained by substituting \( t = 0, 0.2, 0.4, \ldots \) in the above equation.

--

Wind Energy Systems by Dr. Gary L. Johnson  
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Several observations can be made from these results. One is that the indicated wind speed does not change substantially between successive readings. This would imply that the data rate is ample, so that the period between recordings could be increased to perhaps 0.5 or 1.0 s without a serious reduction in data quality.

Another observation is that the mechanical time constant $\tau$ is rather large compared with most electrical time constants of a data collection system. As long as the electrical time constant of an input filter circuit is less than $\tau$, the data system response is limited by the mechanical rather than the electrical filtering. Since $\tau$ decreases with increasing wind speed, it is important to keep the electrical time constant less than the minimum $\tau$ of interest. A value of 0.2 s would probably be acceptable for this type of anemometer.

It can be seen by extending the above table that about 5 mechanical time constants, or about 8 s in this case, are required for the anemometer to reach its new equilibrium angular velocity. If this time is too long for some application, then a smaller anemometer with faster response would need to be used.

Rotating anemometers tend to accelerate faster in a positive gust than they decelerate in a negative gust, making their average output somewhat high in gusty winds. This is one of the effects not correctly represented by Eq. 4. This error is difficult to measure but could be as much as 10 per cent in very gusty winds.

Another type of rotational anemometer is the contact anemometer. Instead of an electrical generator or a slotted disk on the rotating shaft, there is an input to a gear-reduction transmission. The transmission output makes one revolution for a fixed number of revolutions of the cup wheel or propeller, which is correlated to the amount of air passing the anemometer. A contact is operated once per revolution, which when supplied from a voltage source will cause a short pulse to be delivered to a recording device, typically a strip chart recorder or a totaling counter. If a contact closure occurs for every mile of wind passing the cups, the number of pulses in one hour will give the average wind speed for that hour. If a strip chart traveling at constant speed is used, higher wind speeds are represented by pulses with closer spacing. The two pulses which have the smallest spacing in any time interval can be used to determine the fastest mile for that interval. If pulses are 2 minutes apart for a 1 mile contact anemometer, the average wind speed during that interval was 1 mile per 2 minutes or 30 miles per hour. The instantaneous wind speed would be higher than 30 mph during a portion of the interval, of course, but such fluctuation could not be displayed by a contact anemometer since it is basically an averaging instrument. The fastest mile during a day or a month is one of the items recorded by the National Weather Service, as mentioned in the previous chapter.

A wide variety of gear ratios between the cup wheel or propeller and the contact are used for various applications. The gear ratio is usually expressed as the amount of wind required to pass the anemometer to produce one pulse, e.g. a 1/15 mile contact anemometer produces one pulse for every 1/15 mile of air passing by, or 15 pulses per mile of air passing by.

Contact anemometers connected to battery operated electromechanical counters are able
to rank prospective wind sites with minimum cost and excellent reliability. The site with the highest count at the end of the time period of observation would have the highest average wind speed, and would be expected to be the best site for wind power production. If more detailed information about diurnal and directional variation of wind speed is needed, a more sophisticated data acquisition system can be placed at each site which appeared promising in the initial screening.

4 OTHER ANEMOMETERS

Pressure Plate Anemometer

Another type of anemometer is known as the pressure plate or normal plate anemometer[6]. This is the oldest anemometer known, having been developed by Robert Hooke in 1667. It uses the fact that the force of moving air on a plate held normal to the wind is

\[ F_w = cA \rho \frac{u^2}{2} \text{ N} \]  

(14)

where \( A \) is the area of the plate, \( \rho \) is the density of air, \( u \) is the wind speed, and \( c \) is a constant depending on the size and shape of the plate but not greatly different from unity. This force can be used to drive a recording device directly or as input to a mechanical to electrical transducer. The main application of this type of anemometer has been in gust studies because of its very short response time. Gusts of duration 0.01 s can be examined with this anemometer.

This type of anemometer may become a serious competitor of the rotating anemometer if an inexpensive, reliable mechanical to electrical transducer is developed. If a cylinder is used instead of a flat plate it would be possible to have an anemometer with no moving parts. This would eliminate many maintenance problems and sources of error. Experimental anemometers like this have been built using strain gauges but have not performed satisfactorily. A mechanically stiff cylinder has been used which tends to vibrate or oscillate in the wind and make measuring the wind speed difficult. Strain gauges require power to operate, which is a disadvantage in remote sites, and also present problems in building and installing so that good results over the full range of wind speeds are obtained. Until some sort of breakthrough is made in the technology, this type of anemometer will see relatively little use.

Pressure Tube Anemometer

Yet another type of anemometer is the pressure tube anemometer. It is not used much in the field because of difficulties with ice, snow, rain, and the sealing of rotating joints. However, it is often used as the standard in a wind tunnel where these difficulties are not present. It has been known for almost two centuries that the wind blowing into the mouth of a tube causes
an increase of pressure in the tube, and that an air flow across the mouth causes a suction. The pressure in a thin tube facing the wind is

\[ p_1 = p_s + \frac{1}{2} \rho u^2 \quad \text{Pa} \quad (15) \]

where \( p_s \) is the atmosphere pressure. The pressure in a tube perpendicular to the wind is

\[ p_2 = p_s - \frac{1}{2} c_1 \rho u^2 \quad \text{Pa} \quad (16) \]

where \( c_1 \) is a constant less than unity. The total difference of pressure will then be

\[ \Delta p = p_1 - p_2 = \frac{1}{2} \rho u^2 (1 + c_1) \quad \text{Pa} \quad (17) \]

The combination of parallel and perpendicular tubes is preferred because the atmospheric pressure term can be eliminated. The pressure difference can be measured with a manometer or with a solid state pressure-to-voltage transducer. Air density needs to be measured also, in order to make an accurate computation of \( u \) in Eq. 17.

This technique of measuring wind speed also lends itself to a fully solid state measurement system with no moving parts. A number of perpendicular pairs of pressure tubes would be required, each with its own differential pressure transducer feeding into a microprocessor. The microprocessor would select the transducer with the largest value, perhaps correct for pointing error, compute air density from atmospheric pressure and temperature measurements, and compute the wind speed. The major difficulty would be keeping the pressure tubes free of moisture, dirt, and insects so that readings would be accurate.

**Hot Wire Anemometers**

The hot wire anemometer depends on the ability of the air to carry away heat. The resistance of a wire varies with temperature, so as the wind blows across a hot wire the wire tends to cool off with a corresponding decrease in resistance. If the hot wire is placed in one leg of a bridge circuit and the bridge balance is maintained by increasing the current so the temperature remains constant, the current will be related to the wind speed by

\[ i^2 = i_o^2 + K \sqrt{u} \quad \text{A}^2 \quad (18) \]

where \( i_o \) is the current flow with no wind and \( K \) is an experimentally determined constant. The wire, made of fine platinum, is usually heated to about 1000°C to make the anemometer output reasonably independent of air temperature. This anemometer is especially useful in measuring very low wind speeds, from about 0.05 to 10 m/s.
Rather sophisticated equipment is required to make the hot wire anemometer convenient to use. Rain drops striking the wire may cause it to break, making it difficult to use outdoors on a continuous basis. The power consumption may also be significant. The hot wire anemometer will probably not be important to wind power studies because of these difficulties.

Doppler Acoustic Radar

Sonic anemometers, or Doppler acoustic radars, as they are often called, use sound waves reflecting off small blobs or parcels of air to determine wind speed. A vertical profile of wind speed is typically determined from one receiving antenna and three transmitting antennas, located at ground level and arranged as in Fig. 8. The receiving antenna is pointed straight up. The transmit antennas are aimed toward the vertical line above the receiving antenna. They need to have rather narrow beamwidths so they do not illuminate the receiving antenna directly but just the space above it. Transmit beamwidths of about 50 degrees in the vertical and 35 degrees in the horizontal can be obtained from commercially available high frequency driver and spectral horn speaker combinations. The receiving antenna may consist of another high frequency driver coupled to a parabolic dish, with the entire antenna inside an acoustic enclosure which serves to suppress the side lobes of the receive antenna pattern. The receive antenna beamwidth may be around 15 degrees with a good enclosure.

![Figure 8: Doppler acoustic radar configuration.](image)

Each transmit antenna in sequence broadcasts an audio pulse in the frequency range of 2000 to 3000 Hz. Some of these signals are reflected by atmospheric scatterers (small regions of slightly different density or pressure in the air stream) into the receiving antenna. The frequency and phase of the received signal are analyzed to determine the horizontal and vertical velocities of the atmospheric scatterers. Time delays are used to map the velocities...
at different heights.

The advantages of such a system are many:

1. No tower is needed.
2. The anemometer has no moving parts.
3. No physical devices interfere with the flow of air being measured.
4. The instrument is best suited for examining heights between 30 and 100 m.
5. Measurements can be made over the entire projected area of a wind turbine.
6. Complex volume measurements can be made.

Disadvantages include the large size and complexity of the antennas and the relatively high cost. There are also problems with reflections from the ground or from nearby towers, especially in complex terrain. If these disadvantages can be overcome in a reliable system, Doppler acoustic radar will be a widely used tool in wind measurements.

The same Doppler effect can be used with microwave radar or with lasers. These systems have the same advantages over conventional anemometers as the acoustic radar, but tend to be even more complex and expensive. The beamwidths are much smaller so a rather small volume can be examined. Mechanical movement of the antennas is then required to examine a larger region, which increases the cost significantly. Unless these problems are solved, the microwave and laser systems will see only limited use.

5 WIND DIRECTION

The wind vane used for indicating wind direction is one of the oldest meteorological instruments. Basically, a wind vane is a body mounted unsymmetrically about a vertical axis, on which it turns freely. The end offering the greatest resistance to the wind goes downwind or to the leeward. The wind vane requires a minimum normal or perpendicular wind speed to initiate a turn. This minimum is called the starting threshold, and is typically between 0.5 and 1 m/s. A wind vane and direction transmitter made by Electric Speed Indicator Company is shown in Fig. 9.

Wind direction is usually measured at some distance from where the information is needed. The most convenient way of transmitting direction information is by electric cable so a mechanical position to electrical output transducer is required. One type of wind direction transmitter which works well for digital data systems is a potentiometer (a three terminal variable resistor), a voltage source $V_B$, and an analog to digital converter, as shown in Fig. 10. The potentiometer is oriented so the output voltage $V_d$ is zero when the wind is from the
north, $V_B/4$ when the wind is from the east, $V_B/2$ when the wind is from the south, etc. The A/D converter filters noise and converts $V_d$ to a digital form for recording. The potentiometer is usually wire-wound so $V_d$ changes in small discrete steps as the wiper arm rotates. The A/D output always changes in discrete steps even for a smoothly varying input. These two effects make direction output resolutions of less than 3 degrees rather impractical, but such a resolution is usually quite adequate.

Each digital number represents a range of wind direction, in a similar manner to the wind speed ranges discussed earlier. This range can be changed by adjusting the voltage $V_B$. Suppose that we have a 7 bit A/D converter, which will have 128 different digital numbers if the input voltage varies from zero all the way to rated voltage. $V_B$ can be adjusted to a value less than the rated voltage of the A/D converter so the A/D output just reaches 120 at the maximum setting of the potentiometer, which makes each digital number, or bin, represent a range of 3 degrees. Each of the eight cardinal directions then can easily be determined by adding 15 adjacent bins to get a total of 45 degrees. This increment size does not work as well for a 16 direction system since this would require splitting a bin. If 16 directions are required, an 8 bit A/D converter with 1.5 degree bins may be desirable.
When the wind is from the north and is varying slightly in direction, the potentiometer output will jump back and forth between 0 and $V_B$. This is not a problem to the A/D or the recording system, although any computer analysis program will have to be able to detect and accommodate this variation. If a mechanical output is desired, such as an analog voltmeter or a strip chart recorder, this oscillation from maximum to minimum and back make the instruments very difficult to read. It is possible to overcome this with a potentiometer system, but other techniques are more commonly used.

If only a mechanical or visual output is required, perhaps the most satisfactory method of indicating angular position at a distance is by the use of self-synchronous motors, or synchros. A synchro consists of a stator with a balanced three-phase winding and a rotor of dumbbell construction with many turns of wire wrapped around the stem. The rotor leads are brought out of the machine by slip rings. The connection diagram is shown in Fig. 11. The transmitter $T$ is mechanically connected to a wind vane while the receiver $R$ is connected to a pointer on an indicating instrument. Both rotors are connected to the same source of ac. Voltages are then induced in the stator windings due to transformer action, with amplitudes that depend on the reluctance of the flux path, which depends on the position of the rotors. When both rotors are in the same angular position, the voltages in the corresponding windings of the stators will be the same. The phase angle of each winding voltage has to be the same because of the common single-phase source. When the induced voltages are the same, and opposing each other, there will be no current flow and no rotor torque. When the transmitter rotor is moved, voltage magnitudes become unbalanced, causing currents to flow and a torque to be produced on the receiver rotor. This causes the receiver rotor to turn until it again is in alignment with the transmitter rotor.

![Figure 11: Synchro wind direction transmitter and receiver.](image)

Synchros are widely used for controlling angular position as well as in indicating instruments. They may have static or dynamic errors in some applications. The receiver often has
a mechanical viscous damper on its shaft to prevent excessive overshoots. Perhaps the major disadvantage is that a synchro operated direction system may cost 50-100% more than a wind speed system of equal quality. For this reason, other remote direction indicating instruments are often used on wind systems.

The direction system used by Electric Speed Indicator Company to give a visual meter reading is shown in Fig. 12. The wind direction transmitter contains a continuous resistance coil in toroid form, around the edge of which move two brushes spaced 180 degrees apart. The brushes are attached to the wind vane shaft and turn with the shaft. A dc voltage of perhaps 12 V is applied to the two brushes. This causes three voltages between 0 and 12 volts to appear at the three equally spaced taps of the resistance coil. These three voltages are then connected to three taps on a toroidal coil located on a circular iron core. A small permanent magnet located at the center of the iron core, and supporting the indicator pointer shaft, follows the magnetic field resulting from the currents through the three sections of the coil, causing the pointer to indicate the direction of the wind. This system performs the desired task quite well, at somewhat less cost than a synchro system.

A sudden change in wind direction will cause the vane to move to a new direction in a fashion described by an equation of motion. There will be some time delay in reaching the new direction and there may be some overshoot and oscillation about this point. These effects need to be understood before a direction vane can be properly specified for a given application[4].

A simple wind vane is shown in Fig. 13. The vane is oriented at an angle $\theta$ with respect to a fixed reference, while the wind is at an angle $\gamma$ with respect to the same reference. The vane is
free to rotate about the origin and the bearing friction is assumed to be negligible. The positive direction for all angles, as well as $d\theta/dt$ and $d^2\theta/dt^2$, is taken to be the counterclockwise direction. The torque $T$ is in the clockwise direction so the equation of motion can be written as

$$I \frac{d^2\theta}{dt^2} = -T = -Fr$$

where $I$ is the moment of inertia, $r$ is the distance from the pivot point to the centroid of the tail and $F$ is the equivalent force of the wind acting at the centroid.

\[ F = cA\rho \frac{u^2}{2} (\beta + \Delta\beta) \]  

where $c$ is a constant depending on the aerodynamics of the vane, $A$ is the area, $\rho$ is the air density, and $u$ is the wind speed. The $\Delta\beta$ term is necessary because of the motion of the vane. If $d\theta/dt$ is positive so the vane is rotating in a counterclockwise direction, the relative motion of the vane with respect to the wind makes the wind appear to strike the vane at an angle $\beta + \Delta\beta$. This is illustrated in Fig. 14 for the case $\beta = 0$. The vane centroid is rotating in the counterclockwise direction at a speed $r\omega$, which when combined with the wind velocity $u$ yields an apparent wind velocity $u'$ at an angle $\Delta\beta$ with respect to $u$. The velocity $u'$ is a vector that shows both the speed and the direction of the wind. If $r\omega$ is small compared with $u$, as will normally be the case, then $\Delta\beta$ can be approximated by

\[ u' \approx u \cos \Delta\beta \]
\[ \Delta \beta = \frac{r \omega}{u} \]  

\[ \frac{d^2 \theta}{dt^2} + \frac{cA\rho ur^2}{2I} \frac{d\theta}{dt} + \frac{cA\rho u^2 r}{2I} \theta = \frac{cA\rho u^2 r}{2I} \gamma \]  

This is a second-order differential equation which is normally rewritten as

\[ \frac{d^2 \theta}{dt^2} + 2\xi \omega_n \frac{d\theta}{dt} + \omega^2_n \theta = f(t) \]  

where the parameters \( \omega_n \) and \( \xi \) are given by

\[ \omega_n = u \sqrt{\frac{cA\rho r}{2I}} \text{ rad/s} \]  

\[ \xi = \sqrt{\frac{cA\rho r^3}{8I}} \]  

The general solution to the homogeneous differential equation \( f(t) = 0 \) is assumed to be

\[ \theta = A_1 e^{s_1 t} + A_2 e^{s_2 t} \]  

where \( s_1 \) and \( s_2 \) are the roots of the characteristic equation.

Figure 14: Effective direction of wind due to vane rotation.

We now use the fact that \( \omega = d\theta/dt \) and \( \beta = \theta - \gamma \) to write Eq. 19 in an expanded form:
\[ s_2 + 2\xi \omega_n s + \omega_n^2 = 0 \]  

These roots are

\[ s_1 = -\omega_n \xi + \omega_n \sqrt{\xi^2 - 1} \]

\[ s_2 = -\omega_n \xi - \omega_n \sqrt{\xi^2 - 1} \]  

We see that the character of the solution changes as \( \xi \) passes through unity. For \( \xi \) greater than unity, the roots are real and distinct, but when \( \xi \) is less than unity, the roots are complex conjugates. We shall first consider the solution for real and distinct roots.

Suppose that the vane is at rest with both \( \theta \) and \( \gamma \) equal to zero, when the wind direction changes suddenly to some angle \( \gamma_1 \). We want to find an expression for \( \theta \) which describes the motion of the vane. Our initial conditions just after the initial change in wind direction are \( \theta(0^+) = 0 \) and \( \omega(0^+) = 0 \), because of the inertia of the vane. Since \( \omega = d\theta/dt \), we see that \( d\theta/dt \) is zero just after the wind direction change. After a sufficiently long period of time the vane will again be aligned with the wind, or \( \theta(\infty) = \gamma_1 \). The latter value would be the particular solution for this case and would have to be added to Eq. 26 to get the general solution for the inhomogeneous differential equation, Eq. 23.

\[ \theta = A_1 e^{s_1 t} + A_2 e^{s_2 t} + \gamma_1 \]  

Substituting the initial conditions for \( \theta \) and \( d\theta/dt \) in this equation yields

\[ 0 = A_1 + A_2 + \gamma_1 \]

\[ 0 = A_1 s_1 + A_2 s_2 \]  

Solving these two equations for the coefficients \( A_1 \) and \( A_2 \) yields

\[ A_1 = \frac{\gamma_1}{s_1 / s_2 - 1} \]

\[ A_2 = \frac{\gamma_1}{s_2 / s_1 - 1} \]
These coefficients can be evaluated for given vane parameters and a particular wind speed and inserted in Eq. 29 to give an expression for \( \theta \) as a function of time.

When the roots are complex conjugates a slightly different general solution is normally used. Equation 29 is valid whether the roots are real or not, but another form of the solution yields more insight into the physical phenomena being observed. This alternative form is

\[
\theta = \gamma_1 + Be^{-\omega_n \xi t} \sin(\omega_d t + \sigma) \quad \text{rad} \tag{32}
\]

where

\[
\omega_d = \omega_n \sqrt{1 - \xi^2} \quad \text{rad/s} \tag{33}
\]

This form of the solution is a sinusoidal function of time with a phase angle \( \sigma \) and an exponentially decaying amplitude, a so-called damped sinusoid. When the initial conditions for \( \theta \) and \( d\theta/dt \) are substituted into this expression, we get two equations involving the unknown quantities \( B \) and \( \sigma \).

\[
0 = \gamma_1 + B \sin \sigma \\
0 = B \omega_d \cos \sigma - B \omega_n \xi \sin \sigma \tag{34}
\]

Solving for \( B \) and \( \sigma \) yields

\[
B = -\frac{\gamma_1}{\sin \sigma} \\
\sigma = \tan^{-1} \frac{\omega_d}{\omega_n \xi} = \tan^{-1} \frac{\sqrt{1 - \xi^2}}{\xi} \tag{35}
\]

When \( \xi \) is zero, the sinusoid of Eq. 32 is not damped but oscillates at a frequency \( \omega_d = \omega_n \). We therefore call \( \omega_n \) the natural frequency of the system. The quantity \( \xi \) is called the damping ratio since it contributes to the decay or damping of the sinusoid. When \( \xi \) is less than unity, we have oscillation, and this is referred to as the underdamped case. When \( \xi \) is greater than unity, we no longer have oscillation but rather a monotonic change of angle from the initial to the final position. This is called the overdamped case. Critical damping occurs for \( \xi = 1 \). For most direction vanes used in meteorological applications, \( \xi \) is well below 1, giving a damped oscillatory response.
$B$ and $\gamma_1$ can be expressed in either degrees or radians as convenient. However, since $\omega_n$ and $\omega_d$ are both computed in units of rad/s, $\sigma$ must be in radians for Eq. 32 to be evaluated properly.

Figure 15 gives plots of $\theta/\gamma_1$ for various values of $\xi$. As $\xi$ increases, the amount of overshoot decreases, and the distance between zero crossings increases. This means that the damped frequency $\omega_d$ is decreasing as $\xi$ is increasing.

Figure 15: Second-order system response to a step input

It is common to define a *natural period* $\tau_n$ and a *damped period* $\tau_d$ from the corresponding radian frequencies.

\[
\tau_n = \frac{2\pi}{\omega_n}
\]

\[
\tau_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{\omega_n \sqrt{1 - \gamma^2}} = \frac{\tau_n}{\sqrt{1 - \gamma^2}}
\]  \hspace{1cm} (36)

The damped period is the time required to go from one peak to the next, or twice the time required to go from one zero crossing to the next.
It is sometimes necessary to determine the damping ratio and the natural period from wind tunnel tests on a particular vane. From a strip chart recording we can easily determine the overshoot $h$ and the damped period $\tau_d$, as shown in Fig. 16. The time at which the maximum occurs can be estimated from the strip chart and can also be computed from Eq. 32 by taking the time derivative, setting it equal to zero, and solving for $t_{\text{max}}$. It can be shown that

$$t_{\text{max}} = \frac{\tau_d}{2}$$

When this value of $t$ is substituted into Eq. 32 we have

$$\frac{\theta}{\gamma_1} = 1 - e^{-\omega_n\xi t_{\text{max}}} \frac{\sin(\pi + \sigma)}{\sin \sigma}$$

$$= 1 + \exp\left(\frac{-\pi \xi}{\sqrt{1 - \xi^2}}\right)$$

Figure 16: Overshoot $h$ and damped period $\tau_d$ of direction vane tested in a wind tunnel.

By comparing Fig. 16 and Eq. 38, we recognize that

$$h = \exp\left(\frac{-\pi \xi}{\sqrt{1 - \xi^2}}\right)$$

which yields the following expression for $\xi$.

$$\xi = \frac{-\ln h}{\sqrt{\pi^2 + (\ln h)^2}}$$
We then use Eq. 36 and the measured damped period to solve for the natural period.

\[ \tau_n = \tau_d \sqrt{1 - \xi^2} \]  

(41)

We can see from Eqs. 24 and 36 that this period, measured in seconds, is inversely proportional to the wind speed. This is somewhat inconvenient in specifying instruments, so a *gust wavelength* \( \lambda_g \) is defined in a manner similar to the distance constant \( d_m \) in the previous section. We multiply the natural period in seconds by the wind speed in m/s to get a gust wavelength \( \lambda_g \) that is expressed in meters and is independent of the wind speed. That is,

\[ \lambda_g = u \tau_n \]  

(42)

**Example**

A Weather Measure Corporation Model W101-P-AC wind vane is placed in a wind tunnel with \( u = 7 \) m/s. The damped period is measured to be 2.62 s and the measured overshoot \( h \) is 0.53. Find the damping ratio \( \xi \), the natural period \( \tau_n \), and the gust wavelength \( \lambda_g \). Also estimate the time required for the output to reach and remain within five percent of the final value.

From Eq. 40 we find the damping ratio:

\[ \xi = \frac{-\ln(0.53)}{\pi^2 + (\ln(0.53))^2} = 0.20 \]

From Eq. 41 the natural period is

\[ \tau_n = 2.62 \sqrt{1 - (0.2)^2} = 2.57 \text{ s} \]

The gust wavelength is then

\[ \lambda_g = u \tau_n = 7(2.57) = 18.0 \text{ m} \]

The time required to reach and remain within five percent of the final value can be estimated from the exponential factor in Eq. 32. When this term, which forms the envelope of the damped sinusoid, reaches a value of 0.05, the sinusoid is guaranteed to be this value or less. Depending on the location of the peaks, the sinusoid may dip below 0.05 sooner than the envelope, but certainly not later. From the above argument we set

\[ e^{-\omega_n \xi t} = 0.05 \]

and find

\[ t = -\frac{\tau_n \ln(0.05)}{2\pi \xi} = 6.1 \text{ s} \]

This value of \( t \) is sometimes called the *response time.*
The damping ratio of commercially available wind vanes is typically within the range of 0.14 to 0.6 while the gust wavelength is normally between 1 and 18 m. The larger, heavier, and more durable wind vanes have longer gust wavelengths and do not respond fully to rapid changes in wind direction. The smaller, lighter, and more delicate wind vanes have shorter gust wavelengths and respond more accurately to rapid changes in wind direction. The most desirable instrument for turbulence and air pollution dispersion studies is one with a damping ratio close to 0.6 and a short gust wavelength. Measurements made at potential wind power sites normally do not require this sort of detail and a heavier, more rugged instrument can be used.

6 WIND MEASUREMENTS WITH BALLOONS

Economic and technical studies of large wind turbines require a knowledge of wind speed and direction throughout at least the first 100 m above the earth’s surface. The possible existence of nocturnal jets makes wind data desirable at even greater heights, up to several hundred meters. Meteorological towers of these heights are very expensive, so alternative methods of measuring wind data at these heights are used whenever possible. One such method which has been widely used is the release and tracking of meteorological balloons. This method allows the relatively fast and inexpensive screening of a number of sites so a meteorological tower can be properly located if one is found to be necessary.

About 600,000 meteorological balloons are released annually at various National Weather Service stations throughout the United States. The majority of these are relatively small and are released without load. They are visually tracked and the wind speed computed from trigonometric relationships. Larger balloons carry payloads of electronic instrumentation which telemeter information back to earth. These can be used in inclement weather where visual tracking is impossible, and also can provide data from much greater heights. The nocturnal jet is primarily a fair weather phenomenon so the smaller balloons are quite adequate for these lower level wind measurements.

A widely used meteorological balloon is the Kaysam 35P, made by the Kaysam Corporation, Paterson, New Jersey. This balloon has a mass of 30 grams and is normally inflated with hydrogen gas, until it just lifts an attached 140 gm mass. The balloon is then said to have a free lift of 140 gm when this mass is removed. The diameter at launch is about 0.66 m with a volume of about 0.15 m$^3$. The balloon expands as it rises, finally bursting at a diameter of about 1.2 m at a height of over 10,000 m. These balloons are available in three colors; white for blue skies, red for broken skies, and black for overcast skies. At night, a candle or a small electric light can be attached to the balloon to aid the visual tracking process.

When the balloon is released, it will accelerate to its terminal vertical velocity in about five seconds. This terminal velocity depends somewhat on the temperature and pressure of the atmosphere but is usually assumed to be the long term average value. Perhaps the major source of error in measuring wind speeds with balloons is an ascent rate different from the
assumed average. This can be caused by turbulent air which contains significant vertical wind speeds. Mountain valleys or hillsides with strong vertical updrafts or downdrafts should be avoided for this reason.

The National Weather Service forms used for recording the 30 gm balloon data indicate a slightly greater ascent rate during the first five minutes of flight than that assumed later in the flight. They show an average rise of 216 m during the first minute, 198 m the second and third minutes, 189 m the fourth and fifth minutes, and 180 m each minute thereafter. The average rate of ascent therefore decreases from about 3.6 m/s near the ground to about 3 m/s at levels above 1000 m.

The instrument used in tracking the balloon after launch is the *theodolite*, a rather expensive instrument used in surveying work. The theodolite is mounted on a three-legged support called a *tripod*. The balloon is observed through a telescope and angles of elevation and azimuth are read from dials at prescribed intervals: usually one minute. The observer must stop at the prescribed time, read two angles, and then find the balloon in the telescope again before the time for the next reading. The number of possible observations per minute is limited to perhaps three or four by the speed of the human operator. This yields average windspeeds over rather large vertical increments: about 200 m with the standard procedure down to about 50 m at the limit of the operator's ability. Even smaller vertical increments are desirable, however, when the height of the lower level of a nocturnal jet is being sought. These can be obtained by attaching a data collection system to a tracker containing two potentiometers mounted at right angles to each other. The basic concept is shown in Fig. 17.

![Figure 17: Block diagram of a simple balloon tracking system](image)

In this system the horizontal potentiometer produces a voltage $V_H$ proportional to azimuth while the vertical potentiometer produces a voltage $V_V$ proportional to elevation. These voltages are *multiplexed* into an analog-to-digital converter and recorded in the memory of a microcomputer. A *multiplexer* is an electronic switching device which connects one input line at a time to the single output line. These values can then be manually recorded on paper after the launch is complete, if a very simple system is desired. More sophisticated electronics are possible, but one needs to remember that portability, ruggedness, and insensitivity to weather extremes are very desirable features of such a system. It should be mentioned that theodolites can be purchased with these potentiometers, but acceptable results can be obtained with a simple rifle scope connected to two potentiometers, at a small fraction of the cost of a theodolite[3].
The voltages $V_H$ and $V_V$ produce integer numbers $N_H$ and $N_V$ at the output of the A/D converter. The actual angles being measured are proportional to these numbers. That is, the azimuth angle $\alpha$ is given by

$$\alpha = kN_H$$

where $k$ is a constant depending on the construction of the potentiometer and the applied battery voltage. A similar equation is valid for the elevation angle $\beta$.

The basic geometry of the balloon flight is shown in Fig. 18. At some time $t_i$ the balloon is at point $P$, at an azimuth angle $\alpha_i$ with respect to the reference direction (the $x$ axis) and an elevation angle $\beta_i$ as seen by the balloon tracker located at the origin. The balloon is at a height $z_i$ above the horizontal plane extending through the balloon tracker. The vertical projection on this plane is the point $P'$. At some later time $t_j$ the balloon has moved to point $Q$, at angles $\alpha_j$ and $\beta_j$, and with projection $Q'$ on the horizontal plane. The length $P'Q'$ represents the horizontal distance traveled during time $\Delta t = t_j - t_i$. The average wind speed during the time $\Delta t$ is

$$u_{ij} = \frac{P'Q'}{\Delta t}$$

If the vertical distance $z_i$ is known from the balloon ascent rate and the time elapsed since launch, the distance $r_i$ can be computed from

$$r_i = z_i \cot \beta_i$$
The projection of the balloon flight on the horizontal plane is shown in Fig. 19. The distance \( A_i \) is given by

\[
A_i = r_i \cos \alpha_i = z_i \cot \beta_i \cos \alpha_i
\]  

(46)

The distance \( B_i \) is given by a similar expression:

\[
B_i = r_i \sin \alpha_i = z_i \cot \beta_i \sin \alpha_i
\]  

(47)

The projections of \( P'Q' \) on the \( x \) and \( y \) axes are given by

\[
\Delta A = A_j - A_i = z_j \cot \beta_j \cos \alpha_j - z_i \cot \beta_i \cos \alpha_i
\]  

(48)

\[
\Delta B = B_j - B_i = z_j \cot \beta_j \sin \alpha_j - z_i \cot \beta_i \sin \alpha_i
\]  

(49)

The length \( P'Q' \) is then given by

\[
P'Q' = \sqrt{(\Delta A)^2 + (\Delta B)^2}
\]  

(50)

Figure 19: Projection of balloon flight on a horizontal plane.

The angle \( \delta \), showing direction of balloon travel with respect to the \( x \) axis, is given by

\[
\delta = \tan^{-1} \frac{\Delta B}{\Delta A}
\]  

(51)

An alternative approach is to combine the length and direction into a vector quantity \( P'Q' \), where, by using complex arithmetic, we have
\[ P'Q' = \Delta A + j \Delta B = P'Q' \angle \delta \]  

(52)

In general, \( \Delta A \) may be either positive or negative, and the same is true for \( \Delta B \). Some care needs to be exercised if \( \delta \) is to be computed from Eq. 51 using a hand calculator because the inverse tangent function normally gives a result between \(-90^\circ\) and \(+90^\circ\). This does not cover the full \( 360^\circ \) required in this application so a sketch must be drawn of the actual setting each time, and the correct angle determined. Those hand calculators which have a rectangular to polar conversion will normally give both \( P'Q' \) and \( \delta \) correctly from Eq. 52 under all circumstances without additional computations.

The balloon tracker needs to be located so the wind is blowing at nearly a right angle across the launch point, for maximum accuracy. The balloon tracker then needs to be oriented on the tripod so the azimuth potentiometer will stay in its linear region during the balloon flight. This means that the \( x \) axis of Figs. 18 and 19 is arbitrarily oriented at some angle \( \alpha_a \) with respect to north. This is shown in Fig. 20. The angle \( \delta \) shows the direction of balloon flight with respect to the \( x \) axis. What is really needed, however, is the direction \( \theta \) the wind is coming from, with respect to north. As implied in the previous section, a wind from the north is characterized by \( \theta = 0^\circ \) or \( 360^\circ \), a wind from the east by \( \theta = 90^\circ \), etc. The final result for wind direction, after \( \delta \) is computed and \( \alpha_a \) is measured at a particular site, is

\[ \theta = 180 - (\delta - \alpha_a) \]  

(53)

Figure 20: Geometry for computing wind direction.

Example

A balloon launched at Manhattan, Kansas yielded the following values of \( N_H \) and \( N_V \) from a simple balloon tracker at 10-s intervals:
The horizontal distance between the balloon tracker and the launch point was 90 m. The constant $k$ was 0.48 for both elevation and azimuth angles. The angle $\alpha_a$ was determined to be $140^\circ$. The average ascent rate for the first minute may be assumed to be 3.6 m/s. Prepare a table showing wind speed and direction as a function of time.

The elevation angle to the launch point is given by

$$\beta_o = kN_V = (0.48)(6) = 2.88^\circ$$

We then find the initial height from Eq. 45.

$$z_o = \frac{r_o}{\cot \beta_o} = \frac{90}{\cot 2.88^\circ} = 4.5 \text{ m}$$

The initial azimuth angle is

$$\alpha_o = kN_H = (0.48)(2) = 0.96^\circ$$

The height of the balloon after the first 10 s of travel will be

$$z_1 = 4.5 + 10(3.6) = 40.5 \text{ m}$$

The other desired quantities are

$$A_o = r_o \cos \alpha_o = 90 \cos 0.96^\circ = 90 \text{ m}$$

$$B_o = r_o \sin \alpha_o = 90 \sin 0.96^\circ = 1.5 \text{ m}$$

$$\alpha_1 = (0.48)(58) = 27.84^\circ$$

$$\beta_1 = (0.48)(47) = 22.56^\circ$$

$$A_1 = z_1 \cot \beta_1 \cos \alpha = 40.5(2.41)(0.88) = 86.2 \text{ m}$$

$$B_1 = 40.5(2.41)(.47) = 45.6 \text{ m}$$
\[ \Delta A = A_1 - A_0 = 86.2 - 90 = -3.8 \text{ m} \]

\[ \Delta B = B_1 - B_0 = 45.6 - 1.5 = 44.1 \text{ m} \]

\[ P'Q' = \Delta A + j \Delta B = -3.8 + j44.1 = 44.3 \angle 95^\circ \]

\[ u_{01} = \frac{P'Q'}{\Delta t} = \frac{44.3}{10} = 4.43 \text{ m/s} \]

\[ \theta = 180 - (\delta - \alpha_a) = 180 - (95 - 140) = 225^\circ \]

Proceeding in a similar manner for the other heights yields the following table:

<table>
<thead>
<tr>
<th>Time</th>
<th>i</th>
<th>( \alpha_i )</th>
<th>( \beta_i )</th>
<th>( z_i )</th>
<th>( A_i )</th>
<th>( B_i )</th>
<th>( u_{ij} )</th>
<th>( \theta_{ij} ) (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0.96</td>
<td>2.88</td>
<td>4.5</td>
<td>90</td>
<td>1.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>27.84</td>
<td>22.56</td>
<td>40.5</td>
<td>86.2</td>
<td>45.6</td>
<td>4.43</td>
<td>225</td>
</tr>
<tr>
<td>20</td>
<td>2</td>
<td>60.0</td>
<td>25.44</td>
<td>76.5</td>
<td>80.4</td>
<td>139.3</td>
<td>9.39</td>
<td>226</td>
</tr>
<tr>
<td>30</td>
<td>3</td>
<td>74.88</td>
<td>24.0</td>
<td>112.5</td>
<td>65.9</td>
<td>243.9</td>
<td>10.56</td>
<td>222</td>
</tr>
<tr>
<td>40</td>
<td>4</td>
<td>81.60</td>
<td>22.56</td>
<td>148.5</td>
<td>52.2</td>
<td>353.6</td>
<td>11.06</td>
<td>223</td>
</tr>
<tr>
<td>50</td>
<td>5</td>
<td>85.44</td>
<td>21.12</td>
<td>184.5</td>
<td>38.8</td>
<td>476.1</td>
<td>12.32</td>
<td>224</td>
</tr>
</tbody>
</table>

We see that in the first interval that the average wind speed was 4.43 m/s while the second interval showed an average speed of 9.39 m/s, with the speeds building to 12.32 m/s in the fifth interval. It would appear from these data that a nocturnal jet was starting at about 40 or 50 m above the balloon tracker. A large wind turbine with hub height of perhaps 80 m would be operating at near capacity conditions, even though the winds predicted by an anemometer at 10 m elevation would perhaps not be adequate to even start the turbine.

It should be mentioned that balloon data normally give more erratic wind speeds than those shown in this example. After the first few intervals, the angular change is rather small, and the discrete nature of the A/D output will tend to make uniform wind speeds appear larger in one interval and smaller in the next. A limited amount of empirical averaging or smoothing of the calculated wind speeds may therefore be necessary if a monotonic curve is desired.

7 PROBLEMS

1. An eight bit A/D converter which reaches maximum output for an input of 5 V is connected to the anemometer of Fig. 4. Specify the range of wind speeds represented by the four digital outputs 0, 1, 2, and 255. Show wind speeds to at least two decimal places, in m/s.
2. The distance constant for the Climet Instruments Model WS-011 anemometer is 1.5 m. Equilibrium has been reached at a wind speed of 4 m/s when the wind speed suddenly increases to 7 m/s. How long does it take for the indicated wind speed to reach 6.9 m/s?

3. The distance constant for the Climatronics Model WM-III anemometer is 2.5 m. Equilibrium has been reached at 5 m/s when the wind speed suddenly increases to 9 m/s, remains there for one second, and then decreases suddenly to 6 m/s, as shown in Fig. 21. Assume that the indicated wind speed after one second, \( u'_0 \), is the new equilibrium speed for the time period denoted by \( t' \). Assume that the anemometer has the voltage output of Fig. 4, and that the A/D converter of Problem 1 is connected to the anemometer and is sampling the voltage every 0.2 s starting at \( t = 0 \). Make a table of the expected digital output of the A/D converter (in decimal form) from time \( t = 0 \) to \( t = 2 \) s. Sketch the indicated wind speed for this time range. Does this sampling rate appear adequate to detect and represent gusts of this height and duration? Discuss.

4. The U.S. Corps of Engineers has recorded a considerable amount of wind speed data using a 1/15 mile contact anemometer and a summation period of 5 minutes. What is the average wind speed in m/s during a 5 minute period for which the count was 18?

5. A pressure plate anemometer has a plate 0.1 m on a side. The constant \( c \) may be taken as unity and the atmosphere is at standard conditions (0°C and 101.325 kPa).

   (a) What is the force on the plate at wind speeds of 2, 5, 10, 20, and 40 m/s? If the output signal is proportional to force, comment on the difficulty of building a meter which will accept all speeds up to 40 m/s and still read accurately in the 2 - 6 m/s range.
(b) (optional - extra credit) The ideal transducer would be one with a mechanical or electrical output proportional to the square root of force so the scale of wind speed would be linear on an indicating meter. How might this be accomplished?

6. An Electric Speed Indicator Company direction vane Model F420C has a damping ratio $\xi = 0.14$ and a gust wavelength $\lambda_g = 17.7$ m. The wind speed is $8$ m/s and equilibrium has been reached when the wind direction suddenly changes by 30°. Evaluate the coefficients of Eq. 32. Plot $\theta$ on a sheet of graph paper for the time period $t = 0$ to $t = 4$ s assuming that $\theta = 0$ at $t = 0$.

7. Repeat Problem 6 for the R.M. Young Company Gill Anemometer - Bivane which has damping ratio $\xi = 0.60$ and a gust wavelength $\lambda_g = 4.4$ m. Plot on the same graph.

8. You develop a new wind vane for which the damping ratio $\xi$ is 1.3 and the gust wavelength $\lambda_g$ is 4.4 m. Repeat Problem 6 for this vane, noting that the appropriate solution is Eq. 29. Plot the result on the same graph as Problems 6 and 7.

9. A balloon launched at Manhattan, Kansas yielded the following values of $N_H$ and $N_V$ at 10 s intervals:

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_V$</td>
<td>9</td>
<td>46</td>
<td>85</td>
<td>94</td>
<td>91</td>
<td>86</td>
</tr>
<tr>
<td>$N_H$</td>
<td>236</td>
<td>218</td>
<td>156</td>
<td>81</td>
<td>41</td>
<td>18</td>
</tr>
</tbody>
</table>

The horizontal distance between the balloon tracker and the launch point was 122 m. The constant $k$ was 0.48 for both elevation and azimuth angles. The angle $\alpha_a$ was determined to be 335°. The average ascent rate for the first minute may be assumed to be 3.6 m/s. Prepare a table showing wind speed and direction as a function of time.

References


